MA 1971 Bridge to Higher Mathematics Name: Solutions

Quiz 5 D Term, 2021

I affirm that I have not consulted my text, notes or any reference, paper or electronic, or any person once I opened and/or looked at this quiz.

Signature:

High: 25
Median: 23
Low: 10

Show all work needed to reach your answers.

1. (5 points) Please give the graph theory definition of a *tree*.

(+1) A tree is a connected graph containing no cycles. (+4)

2. (5 points) Suppose that you have a friend who has taken a graph theory course and who claims to have drawn a tree with 15 vertices and 12 edges. In one sentence, please explain why is your friend mistaken.

(+5) For any tree, $|V| - |E| = 1$; since $15 - 12 = 3$ this graph can not be a tree.

3. (10 points) An edge in a connected graph G is a *bridge* iff its removal will disconnect the graph. Please show that if G is a cycle, then none of its edges is a bridge. Hint: Start with the definition of a cycle.

A cycle is a graph of the form $\{v_0, e_1, v_1, e_2, \ldots e_{n-1}, v_{n-1}, e_n, v_n\}$ (+3) where all edges and vertices are distinct (+2) (except that $v_0 = v_n$)

(+5) Suppose that edge e_k is removed from G. Notice that one can still walk from v_{k-1} (or any other vertex v_j where $j \leq k-1$) to v_k (or any other vertex v_i where $i \geq k$) by moving through vertex $v_0 = v_n$. Since one can still move vertices v_j and v_i with either $i, j < k$ or $i, j > k$ without passing through $v_0 = v_n$, removing e_k does not disconnect the graph, no matter which e_k was removed. Thus no edge is a bridge.