MA1971 Name: Solutions

Quiz 3 **D Term, 2013**

Show all work needed to reach your answers.

High: 20
Median: 15
Low: 6

1. (10 points) Please show that

$$\sum_{j=1}^{n} j^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Pf (Induction):

2 • Base Case: For $n=1$, $\sum_{j=1}^{1} j^3 = 1 = \left(\frac{1(2)}{2}\right)^2$ ✓

1 • Assume that the formula holds for $n=k$: $\sum_{j=1}^{k} j^3 = \left(\frac{k(k+1)}{2}\right)^2$,

and show that the formula holds for $n=k+1$: 1

$$\sum_{j=1}^{k+1} j^3 \overset{1}{=} \sum_{j=1}^{k} j^3 + (k+1)^3 \overset{1}{=} \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3 = \frac{(k+1)^2}{4}\left[k^2 + 4(k+1)\right] \quad 2$$

$$= \frac{(k+1)^2}{4}(k^2+4k+4) \overset{1}{=} \frac{(k+1)^2(k+2)^2}{4} \overset{1}{=} \left(\frac{(k+1)((k+1)+1)}{2}\right)^2 = \left(\frac{n(n+1)}{2}\right)^2 ✓$$

Thus both steps in the inductive axiom are satisfied, hence the formula holds $\forall n \in \mathbb{Z}^+$.

2. (10 points) Given that A and B are sets, let $A \cap B$ (A **intersect** B) be the subset of A consisting of those elements of A which are also in B. Using one or more of the axioms, please show that $A \cap B$ exists.

Let $Q(x) :=$ "$x \in B$". Then by Axiom III, 3 3

$$A \cap B = \{x \in A \mid Q(x) \text{ is true}\} \quad 3$$

must exist. 1