MA1971 **Name:** ______________________________

Quiz 5 **D Term, 2015**

Show all work needed to reach your answers.

1. (10 points) Your friend is a something-not-mathematics major, and tells you that it is possible to count the real numbers in $[0,1]$ by writing them in binary form (so each number $x_j \in [0,1]$ is of the form $0.x_{j1}x_{j2}x_{j3}...$ where x_{jk} is either 0 or 1). Your friend gives the count as

$$\begin{aligned} x_1 &= 0.x_{11}x_{12}x_{13}x_{14}... \\ x_2 &= 0.x_{21}x_{22}x_{23}x_{24}... \\ x_3 &= 0.x_{31}x_{32}x_{33}x_{34}... \\ x_4 &= 0.x_{41}x_{42}x_{43}x_{44}... \end{aligned}$$

and so forth

Is your friend correct or wrong? Please explain why.

2. (10 points) By the greatest lower bound property of $\mathbb{R}$ (the real numbers), a decreasing sequence of real numbers that is bounded below must converge to a limit L which is also a real number. Does a decreasing sequence of rational numbers always converge to a rational number? Please give a counterexample to show that $\mathbb{Q}$ (the rational numbers) has no such greatest lower bound property. Hint: Think about the examples we have discussed in class and the homework exercises.