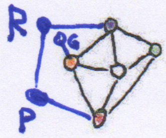
MA1971 Name: Solutions

Quiz 6 D Term, 2015

High: 20
Median: 15
Low: 8

Show all work needed to reach your answers.

1. (10 points) Please explain why a vertex of degree four in a supposed minimal 5-colorable graph in fact forces the graph to be 4 colorable. That is, why can we necessarily change the color of one of the four adjacent vertices to free-up a color of the vertex of degree four?



Consider the red vertex; switch its color to purple. If there is not a red-purple chain connecting this red vertex to the purple vertex at the top of the diagram, we are done. +4 If there is, then consider the orange-green chain leading from the orange vertex. Since the red-purple chain cuts this chain from the green vertex on the right, we can flip the colors on this orange-green chain, +4 guaranteeing that the degree-four vertex can be colored orange. +2

2. (10 points) Consider a connected planar graph with 16 vertices all of which have degree 4. How many regions will this graph divide the plane into (remember that the outer region counts as a region)? Please justify (i.e., prove) your answer.

Because the graph is connected and planar, the Euler formula applies, +2 meaning that $|V| - |E| + |R| = 2$. +3 But also $4|V| = \sum_{k=1}^{16} \deg(v_k) = 2|E|$ +3 (because all the vertices have degree 4. +1 So $|R| = 2 - |V| + |E| = 2 - 16 + 32$ +1

$\Rightarrow |R| = 18$. QED.