Show your work in the space provided. Unsupported answers may not receive full credit.

1. Find the general solutions of the following differential equations. Where initial conditions are given also solve the particular initial value problem.

(a) $y'' + 4y' + 5y = 0$
(b) $y'' + 6y' + 9y = 0$
(c) $y'' - 4y' + 3y = 0$ with $y(0) = 5$, $y'(0) = -3$

2. Suppose that the roots of the characteristic equation for the homogeneous second order ODE with constant coefficients $y'' + p_1 y' + p_0 y = 0$ are the real numbers $r_1$ and $r_2$, with $r_1 \neq r_2$. Use the Wronskian to show that all solutions of the ODE have the form

$$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$$

3. Describe our basic existence and uniqueness result for a single differential equation of the form $y' = f(y, x)$. Include the conditions that $f$ must satisfy to guarantee that the solution through a point $(a, b)$ exists and is unique.

4. Consider the following differential equation.

$$y' = \frac{xy}{x^2 - 1}$$

Solve the IVP with $y(0) = 1$ and determine the interval of existence of your solution.

5. Show that the following ODE is exact, and find the solution.

$$(y \cos(x) + 2xe^y) \, dx + (\sin(x) + x^2e^y - 1) \, dy = 0$$

6. Solve the following ODE.

$$xy' + 2y = \sin(x)$$

7. Consider the ODE

$$y'' + \frac{1}{x} y' - \frac{1}{x^2} y = 0$$

for $x > 0$.

(a) Show that there is a solution of the form $x^r$, where $r$ is a constant.

(b) Find two linearly independent solutions for $x > 0$ and prove that they are linearly independent.