1. Solve the following initial value problem. Assume \(-\pi/2 < x < \pi/2\).

\[ y' + \tan(x)y = 2x \cos(x), \quad y(0) = 2 \]

2. Consider the following differential equation.

\[ y'' - 4y' + 4y = (3 + x)e^{2x} \]

(a) Find the general solution to the homogeneous equation.

(b) Find a particular solution to the non-homogeneous equation. (Hint - write the equation in \(L(D)\) form and use the theorem to simplify the calculation.)

3. Use the Laplace transform to solve the following initial value problem.

\[ y'' + 3y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = -4 \]

4. Consider the system of differential equations \(x' = Ax\) where \(A\) is the matrix given below.

\[ A = \begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix} \]

(a) Find the general solution.

(b) Use your general solution to solve the initial value problem with initial condition \(x_1(0) = 1, \ x_2(0) = 4\).

(c) Classify the origin and sketch the phase plane for this system. Include at least six trajectories in your sketch.

5. Answer the following questions.

(a) Give the two conditions that a linear operator \(L\) must satisfy.

(b) Suppose that a \(2 \times 2\) matrix \(A\) has eigenvalues \(-1 \pm 2i\). Classify the origin and sketch a possible phase plane for the system of differential equations \(\mathbf{x}' = A\mathbf{x}\).

(c) Describe our basic existence and uniqueness result for a single, first order differential equation \(y' = F(x, y)\).