Exam given in term E97, July 24, 1997

1. Solve the initial value problems

    (a) $x'' - 5x' + 6x = 0, \qquad x(0) = 0,\ x'(0) = -1.$

    (b) $x'' + 4x = 8\sin(2t), \qquad x(0) = 0,\ x'(0) = 0.$

2. For the differential equation $x'' + 6x' + 13x = 4\sin(2t)$

    (a) Find the general solution to this equation.

    (b) What part of the solution will become unimportant after a long time (the transient)? What part of the solution will be important after a long time (the limiting behavior)?

    (c) Find the amplitude and period of the limiting solution

3. Write a differential equation that models a spring-mass system with mass 1 kg, damping coefficient 5 kg/s, and a spring constant 4 N/m. Is the system underdamped or overdamped? Using the same damping coefficient and spring constant, what mass will make the system critically damped?

4. For the first-order equation $y' = y^2 - 3y$ where are the equilibria? For each equilibrium, determine if it is stable or if it is unstable.