Find the general solution to the differential equation

$$\frac{dT}{dt} + 0.1T = 0.25t$$

Solution using an integrating factor

Multiply bu μ:

$$\mu T' + 0.1\mu T = 0.25\mu T$$

We want to choose μ so that

$$\mu T' + 0.1\mu T = \mu T' + \mu' T = (\mu T)'$$

Therefore

$$\mu' = 0.1\mu$$

One solution to this differential equation is $\mu = e^{0.1t}$.

We now have

$$(e^{0.1t}T)' = 0.25e^{0.1t}$$

Integrating,

$$e^{0.1t}T = \int 0.25e^{0.1t}s\,dt, \quad \text{or}$$

$$e^{0.1t}T = \int_a^t 0.25e^{0.1s}s\,ds + C$$

So the general solution is

$$T_g = e^{-0.1t}\int_a^t 0.25e^{0.1s}s\,ds + Ce^{-0.1t}$$