Find the general solution to the differential equation

$$\frac{dT}{dt} + 0.1T = 0.25t$$

Solution using Variation of Parameters

A homogeneous solution is

$$T_h = e^{-0.1t}.$$

So we look for a particular solution of the form

$$T_p = v(t)e^{-0.1t}.$$

Substituting,

$$T_p' + 0.1T_p = v'(t)e^{-0.1t} - 0.1v(t)e^{-0.1t} + 0.1v(t)e^{-0.1t}$$

and the last two terms cancel. So v must satisfy

$$v'(t)e^{-0.1t} = 0.25t$$

$$v'(t) = e^{0.1t} \cdot 0.25t$$

$$v(t) = 0.25 \int e^{0.1t} t \, dt \quad \text{or} \quad v(t) = 0.25 \int_a^t e^{0.1s} s \, ds$$

where a is an arbitrary constant.

So the particular solution is

$$T_p = 0.25e^{-0.1t} \int_a^t e^{0.1s} s \, ds$$

and the general solution is

$$T_g = 0.25e^{-0.1t} \int_a^t e^{0.1s} s \, ds + Ce^{-0.1t}$$