TOPICS

- Numerical Methods
  - $\hookrightarrow$ Follow the Slope
  - $\hookrightarrow$ Slope in the Middle?

REMINDER:

Re-Test is TODAY at 4:30 in SL115

- Same rules as the original test
- Maximum score = 90
- If you take the re-test, you keep the score.
- Please SIGN UP after lecture today!

If you score less than 50 on Test 1, DEFINITELY take the Re-Test

Numerical Methods

The definition of the derivative

$$\frac{y(t+\Delta t)-y(t)}{\Delta t} \approx \frac{dy}{dt} = f(t,y)$$

a simple "one-step" formula:

$$y(t+\Delta t) \approx y(t) + f(t,y)\Delta t$$

In words:

$$\text{(New Value)} = \text{(Old Value)} + \text{(Slope)} \times \text{(time-step)}$$

Euler's Method:

Choose a stepsize Δt.

- $t_{k+1} = t_k + \Delta t$
- $y_{k+1} = y_k + f(t_k, y_k)\Delta t$

Start at the initial time t_0 and initial position y_0.

Examples:

Two steps in a picture. . .

Examples:

Go to the Web. . .

Another View of Euler:

Remember Taylor?

$$\begin{aligned} y(t+\Delta t) &= y(t) + \frac{dy}{dt}(t)(\Delta t) \\ &\quad + \frac{d^2y}{dt^2}(c)(\Delta t)^2 \end{aligned}$$

where c is somewhere between t and $t + \Delta t/$

So the second derivative is important?

The Trouble with Euler:

$\rightarrow$ You took the slope on the left. $\leftarrow$

Start at the initial time t_0 and initial position y_0.

Choose a stepsize Δt.

- $t_{k+1} = t_k + \Delta t$
- $y_{k+1} = y_k + f(t_k, y_k)\Delta t$

IDEA: Use the "average" slope

Heun's Method

Start at the initial time t_0 and initial position y_0.

Choose a stepsize Δt.

- $t_{k+1} = t_k + \Delta t$
- $\text{Slope}_1 = f(t_k, y_k)$

  $\text{Slope}_2 = f(t_{k+1}, y_k + \text{Slope}_1 \Delta t)$
- $y_{k+1} = y_k + \left[\dfrac{\text{Slope}_1 + \text{Slope}_2}{2}\right] \Delta t$

Heun's Method

Go to the Web...again.

HW #5: Due Tuesday in Conference

Sec 3.1.8: 1, 2, 7, 41 (Maple or Excel or...;

Sec 3.2.1: 6, 8, 11;