TOPICS

- The Road Ahead...
- A Little Error Analysis
- Steady States and Stability

Linearize!

The Road Ahead...

- Some Numerical Analysis...
- Graphing Stuff...

  Direction Fields, Steady States, Stability, Linearization,...
- Models for Oscillating Systems

  Springs and Masses, Pendulum, Multiple Species, ...
- Analytical Tools...

  Focus on linear models...
- Systems of Differential Equations...

  A little Linear Algebra ...

Which is Better:

Euler? ———————— Heun?

How Much Better?

As Δt goes to zero, the error should go to zero.

Measure the error:

- Euler: $Error \approx (\Delta t)^1$ as Δt goes to 0
- Heun: $Error \approx (\Delta t)^2$ as Δt goes to 0

Euler's Method:

Choose a stepsize Δt.

- $t_{k+1} = t_k + \Delta t$
- $y_{k+1} = y_k + f(t_k, y_k)\Delta t$

Start at the initial time t_0 and initial position y_0.

Heun's Method

Start at the initial time t_0 and initial position y_0.

Choose a stepsize Δt.

- $t_{k+1} = t_k + \Delta t$
- $\text{Slope}_1 = f(t_k, y_k)$

  $\text{Slope}_2 = f(t_{k+1}, y_k + \text{Slope}_1 \Delta t)$
- $y_{k+1} = y_k + \left[\dfrac{\text{Slope}_1 + \text{Slope}_2}{2}\right]\Delta t$

Local or Global?

Local Error $\Longrightarrow$ error made in one timestep.

Global Error $\Longrightarrow$ error made at time T,

$$N = \frac{T}{\Delta t} \text{ timesteps}$$

$$\text{Global Error} \quad = \quad \frac{\text{Local Error}}{\Delta t}$$

Local Euler Error:

Remember Taylor's Theorem:

$$y(t_1) = y(t_0) + y'(t_0)\Delta t + \frac{1}{2}y''(\xi)(\Delta t)^2$$

Compare with

$$y_1 = y_0 + f(t_0, y_0) \cdot \Delta t$$

Local Error:

$$|y(t_1) - y_1| = \left|\frac{1}{2}y''(\xi)\right| (\Delta t)^2 \sim (\Delta t)^2$$

Visit Maple:

Analyzing a Steady State

Start with an equation like

$$y'(t) = f(y(t))$$

If $f(y_{ss}) = 0$ for some constant y_{ss}, then

y_{ss} is called a steady state

Just Say: *Constant Solution*

Some Pictures

- Emigration
- Heat Loss
- Logistic Growth

Perturbations

Define a new function $q(t)$ by

$$y(t) = y_{ss} + q(t)$$

Plug it in ...

Find a new differential equation for $q(t)$...

and look at $\lim_{t \to \infty} q(t)$

Example: Logistic Model

$$P' = aP - sP^2$$

has a steady state

$$P_{ss} = \frac{a}{s}$$

Define $q(t)$ by

$$P(t) = P_{ss} + q(t) = \frac{a}{s} + q(t)$$

and plug it in:

$$q' = P' = a\left(\frac{a}{s} + q\right) - s\left(\frac{a}{s} + q\right)^2$$

Linearize It:

$$q' = -aq - sq^2$$

HW #5: Due Tuesday in Conference

Sec 3.1.8: 1, 2, 7, 41 (Maple or Excel or...;

Sec 3.2.1: 6, 8, 11;

Section 3 in the box on Wednesday