TOPICS

- Systems of Equations

  $\hookrightarrow$ Three Pictures

- Sketching Solution Curves

- Complex Conjugate Case

- Begin the Review...

Announcements:

- Review Session:

  $\hookrightarrow$ Tuesday, March 4: 7:00pm in SL 115

- Test II:

  $\hookrightarrow$ Next Wednesday in Lecture

  Closed Book, Closed Notes, No Calculators

  $\hookrightarrow$ Re-Test Thursday at 3:00pm in FLAUD

  Tests will be graded by 4:00pm on Wednesday

  No Conference on Thursday

Three Basic Pictures...

System:

$$\frac{d}{dt}\begin{bmatrix} y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} y(t) \\ z(t) \end{bmatrix}$$

Examples: Sketch some solution curves

You have a system

$$\vec{y}\,' = A \cdot \vec{y}$$

What if the eigenvalues/eigenvector pairs are

$$\lambda_1 = -0.7,\ \vec{v}_1 =< 4, 1 >, \quad \lambda_2 = +3.4,\ \vec{v}_2 =< 3, -2 >?$$

Examples: Sketch some solution curves

What if the eigenvalues/eigenvector pairs are

$$\lambda_1 = -1 + 4\sqrt{2}i, \quad \lambda_2 = \underline{\hspace{3cm}}?$$

System View: Complex Conjugate Case

System:

$$\frac{d}{dt}\begin{bmatrix} y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 4 & -5 \end{bmatrix} \cdot \begin{bmatrix} y(t) \\ z(t) \end{bmatrix}$$

CE:

$$\det\left(rI - A\right) \begin{bmatrix} r-3 & -4 \\ -4 & r+5 \end{bmatrix} = (r-3)(r+5) - 16 = 0$$

Roots:

Solutions: Go directly to...

$$y_1(t) = e^{-t}\sin(4\sqrt{2}\ t), \quad y_2(t) = e^{-t}\cos(4\sqrt{2}\ t)$$

Find $z_1(t)$ and $z_2(t)$

Add Initial Data:

$$y(0) = 3 \quad z(0) = 1$$

Questions:

Start with

$$\frac{d}{dt}\begin{bmatrix} y(t) \\ z(t) \end{bmatrix}' = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} y(t) \\ z(t) \end{bmatrix}$$

What conditions on a, b, c, and d guarantee that the solutions

- are transient?
- are (only) decaying exponentials?
- are closed curves?
- spiral into the origin?

Partial Answers...

Test II: Guaranteed Problems:

1. Solve a second-order equation
   - $\hookrightarrow$ Homogeneous Solution
   - $\hookrightarrow$ Particular Solution
   - $\hookrightarrow$ Initial Data
2. Analyze an oscillating sytem
3. Solve a system of linear equations

- **HW #10: Due Monday in Lecture**

  Sec 8.2.6: 1, 3, 13, 15, 16 ;

  Sec 8.3.5: 1, 3, 4, 17 ;