TOPICS

- More Models
  - $\hookrightarrow$ Heat Loss
  - $\hookrightarrow$ Epidemics...Systems
- Linear Equations
  - $\hookrightarrow$ A Little Theory
  - $\hookrightarrow$ General Solution

Heat Loss Model

$$\frac{dT}{dt} = -\frac{Ak}{cm}\left(T(t) - T_{\text{out}}\right)$$

Main Ingredients:

(1) Balance Law for Heat Energy Q

(2) Experimental Facts

$\star$ Heat Flow Rate is Proportional to $T_{\text{in}} - T_{\text{out}}$

$\star$ Heat Energy is Proportional to T

$\star$ No Internal Heat Source

Epidemic Model

$$
\begin{aligned}
\frac{dS}{dt} &= -\beta I \cdot S \\
\frac{dI}{dt} &= +\beta I \cdot S - \gamma I \\
\frac{dI}{dt} &= +\gamma I
\end{aligned}
$$

Variables:

- $S =$ *Susceptibles* $I =$ *Infectives* , $R =$ *Removed* individuals

Example: Show that $S + I + R$ is constant.

Linear Equations

Equations that you can SOLVE!

General Form:

$$a_1(x)\frac{dy}{dx} + a_0(x)y = f(x)$$

Independent Variable: x

Dependent Variable: $y \quad (= y(x))$

Coefficients: $a_1(x)$ and $a_0(x)$

Forcing Term: $f(x)$

Examples:

- $\dfrac{dP}{dt} = kP - H$

- $\dfrac{dT}{dt} = -\dfrac{Ak}{cm}\left(T(t) - T_{\text{out}}\right)$

Examples:

- $\dfrac{dP}{dt} = 0.012(1 - \sin(t))P - 0.65(1 + \cos(t))$

- $\dfrac{dP}{dt} = aP - sP^2$

Key Property: Superposition

If $y_1(x)$ solves

$$a_1(x)\frac{dy}{dx} + a_0(x)y = f(x)$$

and $y_2(x)$ solves

$$a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

Then $y_1(x) + y_2(x)$ solves ???

Plug it in...

Key Property: Superposition—Backwards

If $y_1(x)$ and $y_2(x)$ BOTH solve

$$a_1(x)\frac{dy}{dx} + a_0(x)y = f(x)$$

Then $y_1(x) - y_2(x)$ solves ???

Plug it in...

Homogeneous Differential Equations

$\longrightarrow$ Zero is a solution. $\longleftarrow$

When is a linear equation homogeneous?

Plug $y \equiv 0$ into

$$a_1(x)\frac{dy}{dx} + a_0(x)y = f(x)$$

Force the forcing term to be zero: $f(x) \equiv 0$:

Homogeneous Equation:

$$a_1(x)\frac{dy}{dx} + a_0(x)y = 0$$

Solve the Homogeneous Equation

$$a_1(x)\frac{dy}{dx} + a_0(x)y = 0$$

Separate and Integrate:

$$y(x) = C\exp\left(-\int \frac{a_0(x)}{a_1(x)}dx\right)$$

Example:

Solve

$$xy' + y = 0$$

Define General Solution

$$a_1(x)\frac{dy}{dx} + a_0(x)y = f(x)$$

The General Solution is

$$y(x) = y_p(x) + Cy_h(x)$$

where

(1) $y_p(x)$ is *any* solution for the ODE.

(2) $y_h(x)$ is a non-trivial solution for the corresponding homogeneous equation.

Example:

$\dfrac{dP}{dt} = kP - H$ has general solution

$$P(t) = \frac{H}{k} + Ce^{kt}$$

Build the Solution

$$\frac{dT}{dt} + \frac{Ak}{cm}T(t) = \frac{Ak}{cm}T_{\text{out}}$$

(1) Solve $\dfrac{dT}{dt} + \dfrac{Ak}{cm}T(t) = 0$

$$T_h(t) = e^{-\frac{Ak}{cm}t}$$

(2) Find a particular solution:

$$T_p(t) = ???$$

REMINDERS:

HW #2: Due Thursday in Conference

$\hookrightarrow$ **Sec 2.1.7: 3d, 7, 13a, 13c**;

$\hookrightarrow$ **Sec 2.2.4: 2(a,c,f), 9**;

$\star$ You should have a project team... $\star$

HW #3: Due Monday in Lecture

Sec 2.3.1: 1a, 5(a,b,c), 9;

Sec 2.4.5: 11, 13;

Sec 4.1.6: 7–10, 38, 40;

+ Go to your favorite non-Math book and bring back an ODE.

You must provide:

(1) definitions of all variables

(2) *brief* explanation of application

(3) a complete reference (text and page)