TOPICS

- The Story So Far ...
- More About Linear Equations

    ↪ Definite/Indefinite

- When Coefficients are Constant...

    ↪ Characteristic Equation

    ↪ Undetermined Coefficients

The Story So Far...

- A Few Models

    ↪ Free Fall, Populations, Heat Loss, Mixing...

- Some Basic Vocabulary

    ↪ ODE, IVP, Solution,...

- Analyzing Models

    ↪ Sketching solution curves...without solution formulas

    ↪ Solving ODEs; Separate and Integrate,

    ↪ Linear $\Longrightarrow$ homogeneous + particular

ASIDE: Definite or Indefinite?

Solve

$$y' + g(x)y = f(x)$$

(1) find the homogeneous solution $y_h(x)$:

$$y_h(x) = \exp\left(-\int g(x)dx\right)$$

(2) find the particular solution $y_p(x)$:

$$y_p(x) = u(x)y_h(x) = \left[\int \frac{f(x)}{y_h(x)}dx\right] y_h(x)$$

ASIDE^2: Fundamental Theorem of Calculus

$$F(x) = \int_a^x f(z)dz \quad \Longrightarrow \quad F'(x) = ???$$

Definite or Indefinite?

Solve

$$4y' + 12xy = 3, \quad y(1) = \pi$$

Step1: find the homogeneous solution $y_h(x)$:

$$y_h(x) = \exp\left(-\int_1^x 3z\,dz\right)$$

(note that $y_h(1) = 1$)

Continue...

Step 2: find the particular solution $y_p(x)$:

$$y_p(x) = u(x)y_h(x) = \left[\int_1^x \frac{3}{4}e^{-\frac{3}{2}z^2}dz\right] y_h(x)$$

(note that $y_p(1) = 0$)

Finish It...

The general solution is

$$y(x) = y_p(x) + Cy_h(x)$$

Now use the initial data:

$$\pi = y(1) = y_p(1) + Cy_h(1) = 0 + C \cdot 1 = C$$

This worked nicely because the integrals started at $x = 1$.

Linear Equations So Far...

- General Solution = Homogeneous + Particular Solution
- Find a Homogeneous Solution
  - $\hookrightarrow$ Separate and Integrate
  - $\hookrightarrow$ Add C.E.
- Find a Particular Solution
  - $\hookrightarrow$ Vary Your Parameters
  - $\hookrightarrow$ Add U.C.

Characteristic Equation

Find the homogeneous solution for

$$ay' + by = f(x)$$

when a and b are constants!

Assume that the solution is $e^{\lambda x}$ and solve for λ:

Plug it in:

$$a\left[e^{\lambda x}\right]' + b\left[e^{\lambda x}\right] = 0$$

Call this the characteristic equation.

Examples:

- $3y' + 5y = e^{-2x}$ .... C.E. $3\lambda + 5 = 0$

- $P' - kP = E$ .... C.E. $\lambda - k = 0$

- $y' + \pi y = x^3 - 2x + 1$ ..... C.E. $\lambda + \pi = 0$

UNDETERMINED COEFFICIENTS

For constant coefficient equations with special forcing functions, you can guess the form of the particular solution.

Examples:

- $3y' + 5y = e^{-2x}$ .... y_p "looks like" e^{-2x}

- $3y' + 5y = \sin(2t)$ .... y_p "looks like" $\sin(2t)$

- $3y' + 5y = x^3 - 2x + 1$ .... y_p "looks like" a cubic

U.C. Details

Find a particular solution for

$$3y' + 5y = e^{-2x}$$

Guess

$$y_p(x) = Ae^{-2x}$$

Plug it in and solve for A:

U.C. Details

Find a particular solution for

$$3y' + 5y = \sin(2t)$$

Guess

$$y_p(t) = A\sin(2t) + B\cos(2t)$$

Plug it in and solve for A and B:

One Bad Example

Find a particular solution for

$$y' + 5y = e^{-5x}$$

Guess $y_p(x) = Ae^{-5x}$ and plug it in:

$$\left[Ae^{-5x}\right]' + 5Ae^{-5x} = -5Ae^{-5x} + 5Ae^{-5x} = 0$$

You cannot solve for A!

Why? The forcing term is a homogeneous solution!

HW #4: Due Thursday in Conference

Sec 4.2.1: 4, 5, 15;

Sec 4.3.1: 3, 16, 17b, 20;

Sec 4.4.5: 1–4, 21;

Do you have a Project Team?

Sign up with PLA!!!

Test #1: Next Wednesday (February 2) in Lecture