TOPICS

- Summary CE and UC
  - $\hookrightarrow$ Constant Coefficients
  - $\hookrightarrow$ Special Forcing Terms
- Qualitative Behavior
  - $\hookrightarrow$ Graph the solution
    (without knowing the solution)
- Thinking about Test #1

Remember: One Bad Case

Find a particular solution for

$$y' + 5y = 3e^{-5x}$$

Trouble: The forcing term is a homogeneous solution!

Vary your parameters:

$$u(x) = \int \frac{f(x)}{y_h(x)} dx = \int \frac{3e^{-5x}}{e^{-5x}} dx = \int 3dx = 3x$$

Question: Do you need to add C?

Finish It:

Particular Solution:

$$y_p(x) = u(x)e^{-5x} = 3xe^{-5x}$$

Check it:

$$\left(3e^{-5x} - 15xe^{-5x}\right) + 15xe^{-5x} = ???$$

General Solution

$$y(x) =$$

Undetermined Coefficients Works

Make a Table:

- $f(x) = e^{\lambda x}$ .... $y_p(x) = Ae^{\lambda x}$
- $f(x) = \sin(\omega t)$ .... $y_p(x) = A\sin(\omega t) + B\cos(\omega t)$
- $f(x) = x^n$ ..... $y_p(x) = A_0 + A_1x^1 + A_2x^2 + \cdots + A_nx^n$

Undetermined Coefficients Fails

Examples:

- $y' - 2y = x^{-2}$
- $3y' + 5y = \tan(\omega t)$
- $xy' + y = 3e^{4x}$

U.C. Details

Find a particular solution for

$$xy' + y = 3e^{4x}$$

Guess

$$y_p(x) = Ae^{4x}$$

Plug it in and solve for A:

$$x4Ae^{4x} + Ae^{4x} = e3e^{4x} \Longleftrightarrow (4x+1)A = 3$$

Graphing Solutions w/o Solution Formulas:

Main Idea: The ODE contains a lot of information.

$$\frac{dy}{dx} = f(x,y)$$

At every point in the (x,y)–plane, you have a slope.

ASIDE:

Solutions for $\dfrac{dy}{dx} = f(y)$ NEVER oscillate.

Example:

Graph several solution curves for

$$y' = y^2 - 1$$

Keys:

- $y(t)$ is <u>constant</u> when $y'(t) = 0 \iff y^2 - 1 = 0$
  $y = \pm 1$: Call these *steady states.*
- $y(t)$ is increasing when $y_p(t) > 0$

  $$\iff y > 1 \text{ or } y < -1$$
- $y(t)$ is decreasing when $y_p(t) < 0$

  $$\iff y^2 < 1 \iff -1 < y < 1$$

Aside #1:

The "solution" for $y' = y^2 - 1$ is

$$y(t) = \tanh(C - t)$$

Would you rather build the graph from this?

Aside #2:

Graph the RHS of the ODE...

Checking Concavity

You need the second derivative.

$$y' = y^2 - 1 \Longrightarrow y'' = ???$$

Implications:

$\hookrightarrow$ Concave up for $y''(t) > 0$

$\hookrightarrow$ Concave down for $y''(t) < 0$

Make a Table:

Use: $y' = y^2 - 1$ $\qquad$ $y'' = 2yy' = 2y(y^2 - 1)$

REMINDERS:

HW #4: Due Thursday in Conference

Sec 4.2.1: 4, 5, 15;

Sec 4.3.1: 3, 16, 17b, 20;

Sec 4.4.5: 1–4, 21;

Project Drafts Reviews on Thursday

Test #1: Next Wednesday (February 2) in Lecture