TOPICS

- Begin Review for Test 1
- Aside: Numerical Methods

Test #1:

1. Closed Book, Closed Notes, No Calculator
2. 50 minutes in class

Test #1: Guaranteed Problems

1. Solve a linear equation.
   - → Find homogeneous, particular, and general solutions.
   - → Use initial data.
   - → "Analyze" the solution.
2. Work with a model.
   - → Build the ODE from scratch.
   - → *Explain why* you do what you do.
3. One "challenging" problem.

(Look at the sample tests on the Web)

Sample Problem:

Analyze following differential equation:

$$y' + 4y = 100t, \; y(0) = 200$$

(a) Find a nontrivial solution to the corresponding homogeneous equation.

(b) Find a particular solution to the original equation.

(c) Give the general solution and find the unique solution satisfying the initial condition.

(d) Describe the behavior of the solution for large values of the independent variable t.

Key Steps?

- CE to find y_h
- UC to find y_p
- general solution
- IC to find the constant.

A Little Work...

What If...

- double the initial position?

- change the forcing term to $200t$?
- change the forcing term to $f(t) = 4\sin(2t) + 3\cos(5t)$?

Another Sample Problem:

Given that

- $L(t)$ solves $y' + t^3 y = 0$ with $L(0) = 5$,

and

- $M(t)$ solves $y' + t^3 y = \cos(2\pi t)$ with $M(0) = -5$,

find the solution, in terms of L and M, for

$$y' + t^3 y = \cos(2\pi t), \quad y(0) = 10 .$$

Numerical Methods

The definition of the derivative

$$\frac{y(t+\Delta t)-y(t)}{\Delta t} \approx \frac{dy}{dt} = f(t,y)$$

a simple "one-step" formula:

$$y(t+\Delta t) \approx y(t) + f(t,y)\Delta t$$

In words:

$$\text{(New Value)} = \text{(Old Value)} + \text{(Slope)} \times \text{(time-step)}$$

Euler's Method:

Choose a stepsize Δt.

- $t_{k+1} = t_k + \Delta t$
- $y_{k+1} = y_k + f(t_k, y_k)\Delta t$

Start at the initial time t_0 and initial position y_0.

Examples:

Use five steps of Euler's method to approximate $P(50)$ for

$$P'(t) = 0.12P(t) - 0.011P^2(t), \quad P(0) = 1.0$$

- Step 0: $\Delta t = 10; \quad P_0 = 1$
- Step 1: $P_1 = P_0 + \left[0.12P_0 - 0.011P_0^2\right]10 = 2.09$
- Step 2: $P_2 = P_1 + \left[0.12P_1 - 0.011P_1^2\right]10 = 4.1175$
- Step 3: $P_3 = P_2 + \left[0.12P_2 - 0.011P_2^2\right]10 = 7.1936$
- Step 4: $P_4 = P_3 + \left[0.12P_3 - 0.011P_3^2\right]10 = 10.1336$
- Step 5: $P_5 = P_4 + \left[0.12P_4 - 0.011P_4^2\right]10 = 10.1980$

Conclude $P(50) \approx P_5 = 10.1980$

Computing

Terminology Attack:

- Ordinary Differential Equation, Model, Initial Data, Parameters,
- Separate and Integrate
- Linear Equation

More Terminology:

- Superposition Principle .... Who Cares?
- Corresponding Homogeneous Equation
- Homogeneous Solution, Particular Solution, General Solution
- Steady State,

For Monday...

- Visit the old exams and review problems
- Work on the Worcester Yeast Company