TOPICS

- Systems of Equations
  - $\hookrightarrow$ Finding a Particular Solution
- When Coefficients are Constant...

  ...Undetermined Coefficients Works!
  - $\hookrightarrow$ Variation of Parameters is Back
- Review Problems

Announcements:

- Review Session:
  - $\hookrightarrow$ Tuesday, March 4: 7:00pm in Perrault Hall
- Test II:
  - $\hookrightarrow$ Next Wednesday in Lecture
  - $\hookrightarrow$ Re-Test Thursday at 3:30pm in Perrault

Tests will be graded by 4:30pm on Wednesday

No Conference on Thursday

Add Forcing Terms

$$\begin{aligned} y' &= ay + bz + f(t) \\ z' &= cy + dz + g(t) \end{aligned}$$

This System is NOT Homogeneous

General Solution:

$$(y(t), z(t)) = C_1\Big(y_1(t), z_1(t)\Big) + C_2\Big(y_2(t), z_2(t)\Big) + \Big(y_p(t), z_p(t)\Big)$$

Example:

$$\begin{aligned} y' &= -2y - 4z + 15t \\ z' &= 4y - 2z + 10 \end{aligned}$$

CE:

$$\det \begin{bmatrix} (-2-r) & -4 \\ 4 & (-2-r) \end{bmatrix} = (-r-2)^2 + 16 = 0$$

Complex Conjugate Roots: $r = -2 \pm 4i$

$$y_1(t) = e^{-2t}\sin(4t), \quad y_2(t) = e^{-2t}\cos(4t)$$

$$z_1(t) = -e^{-2t}\cos(4t), \quad z_2(t) = e^{-2t}\sin(4t)$$

What about the particular solution?

Try Undetermined Coefficients

Guess: $y_p(t) = At + B,\ z_p(t) = C$

Plug it in:

$$\begin{aligned} A &= 2(At + B) + 4(C) + 15t = (2A + 15)t + (2B + 4C) \\ 0 &= -4(At + B) + 2(C) + 10 = -4At + (2C - 4B + 10) \end{aligned}$$

First Equation Requires: $A = -15,\ 2B + 4C = A$, so $2B + 4C = -15$

Second Equation Requires: $A = 0,\ 2C - 4B = -10$

OOPS!

Second Try:

$$y_p(t) = At + B,\ z_p(t) = Ct + D$$

Plug it in:

$$\begin{aligned} A &= -2(At + B) - 4(Ct + D) + 15t \\ &= (-2A - 4C + 15)t + (-2B - 4D) \end{aligned}$$

$$\begin{aligned} C &= 4(At + B) - 2(Ct + D) + 10 \\ &= (4A - 2C)t + (4B - 2D + 10) \end{aligned}$$

First Equation Requires: $2A + 4C = 15,\ 2B + 4D = -A$

Second Equation Requires: $(4A - 2C) = 0,\ 4B - 2D + 10 = C$

Particular Solution

- Solve the system!

$$A = \frac{3}{2},\ B = \frac{-31}{20},\ C = 3,\ D = \frac{2}{5}$$

- Particular Solution is a LINE

$$\Big(y_p(t), z_p(t)\Big) = \left(\frac{3}{2}t - \frac{31}{20},\ 3t + \frac{2}{5}\right)$$

Slope: $\dfrac{3}{(3/2)} = +2$ Point: $(-31/20, 2/5)$

- Homogeneous Solution is Transient

Bonus Problem:

What Conditions on a, b, c, d guarantee that the homogeneous solution will be transient?

$$\begin{array}{ccccccc} y' & = & ay & + & bz & + & f(t) \\ z' & = & cy & + & dz & + & g(t) \end{array}$$

Look at CE:

CE:

$$\det \begin{bmatrix} (a-r) & b \\ c & (d-r) \end{bmatrix} = (a-r)(d-r) - bc = 0$$

Conclusion:

You have the General Solution

$$\begin{aligned}\Big(y(t), z(t)\Big) &= C_1\Big(y_1(t), z_1(t)\Big) + C_2\Big(y_2(t), z_2(t)\Big) \\ &+ \Big(y_p(t), z_p(t)\Big) \\ &= e^{-2t}\Big(C_1 \sin(4t) - C_2\cos(4t), C_1\cos(4t) + C_2\sin(4t)\Big) \\ &+ \left(\frac{3}{2}t - \frac{31}{20},\ 3t + \frac{2}{5}\right)\end{aligned}$$

Guaranteed Problems:

1. Solve a second-order equation
   - $\hookrightarrow$ Homogeneous Solution
   - $\hookrightarrow$ Particular Solution
   - $\hookrightarrow$ Initial Data
2. Analyze a spring-mass sytem
3. Solve a system of linear equations

HW #12: Due Tuesday in Conference

Sec 8.3: 1, 3, 13

Project #2: Due Thursday at Noon in SH202A