TOPICS

- Population Models
  - ↪ Analyze Data
  - ↪ Balance Law
  - ↪ Emigration
  - ↪ Harvesting
- HW #2: Due Thursday

Solve an IVP

$$P'(t) = 0.012P(t), \quad P(1900) = 76.09$$

Separate:

$$\frac{dP}{P} = 0.012dt$$

Integrate:

$$\ln(P(t)) = 0.012t + c \Longrightarrow P(t) = Ce^{0.012t}$$

What is C?

Find the Constant

$$76.09 = P(1900) = Ce^{0.012 \cdot 1900} = Ce^{22.8}$$

So

$$C = \frac{76.09}{e^{22.8}} \approx 9.537 \times 10^{-9}$$

Better Way:

$$P(t) = 76.09\, e^{0.012(t-1900)}$$

U.S. Population

Year	$P(t)$	$D(t)$	$D(t)/P(t)$
1790	3.93	$\cdots$	$\cdots$
1800	5.30	0.164	$\cdots$
1810	7.22	$\cdots$	$\cdots$
1820	9.62	$\cdots$	$\cdots$
1830	12.90	$\cdots$	$\cdots$
1840	17.12	$\cdots$	$\cdots$
1850	23.26	$\cdots$	$\cdots$

U.S. Population

Year	$P(t)$	$D(t)$	$D(t)/P(t)$
1790	3.93	$\cdots$	$\cdots$
1800	5.30	0.164	0.031
1810	7.22	0.216	0.030
1820	9.62	0.284	0.030
1830	12.90	0.375	0.029
1840	17.12	0.518	0.030
1850	23.26	$\cdots$	$\cdots$

Derive the Model

$$D(t) = \frac{P(t+10) - P(t-10)}{20} \quad \text{and} \quad D(t) \approx \frac{dP}{dt}$$

Model:

$$\frac{1}{P(t)} \cdot \frac{dP}{dt} = 0.030$$

Analyze the Model

$$\frac{dP}{dt} = 0.03P(t)$$

General Solution:

$$P(t) = Ce^{(0.03 \cdot t)}$$

Use the Data from 1790 to get the Particular Solution:

$$P(t) = 3.93\ e^{0.03(t-1790)}$$

Year	Actual	Model
1790	3.93	3.93
1800	5.30	5.30
1810	7.22	7.16
1820	9.62	9.66
1830	12.90	13.05
1840	17.12	17.61
1850	23.26	23.77

Model: P(2000) = 2140.2 (million people)

Exponential Growth

Alternate View: Balance Equation

Net Change in Δt = (rate in)$\times\Delta t$ + (rate out)$\times\Delta t$

$\hookrightarrow$ Net Change = $P(t+\Delta t) - P(t)$

$\hookrightarrow$ Rate In = $k_{\text{in}}P(t)$

$\hookrightarrow$ Rate Out = $k_{\text{out}}P(t)$

Transition to ODE

$$[P(t+\Delta t) - P(t)] = k_{\text{in}} P(t)\Delta t - k_{\text{out}} P(t)\Delta t$$

$$\frac{[P(t+\Delta t) - P(t)]}{\Delta t} = (k_{\text{in}} - k_{\text{out}})\, P(t)$$

Take the limit as $\Delta t \to 0$ to get an ODE:

$$\frac{dP}{dt} = kP(t)$$

where $k = k_{\text{in}} - k_{\text{out}}$

Start Harvesting

Same Balance Equation (almost):

$$P(t+\Delta t) - P(t) = [k_{\text{in}} P(t) - k_{\text{out}} P(t) - H]\,\Delta t$$

Divide by Δt and let $\Delta t \to 0$:

$$\frac{dP}{dt} = kP(t) - H$$

Very different picture

HW #2: Due Thursday in Conference

Sec 2.1: 3d, 12a, 12b; **Sec 2.2: 2c, 4**;

• Go to your favorite non-Math book and bring back an ODE. You must provide:

(1) definitions of all variables

(2) brief explanation of application

(3) a complete reference (text and page)