TOPICS

- Improved Numerical Method
  - $\hookrightarrow$ Slope in the Middle?
- MORE About Linear Equations
  - $\hookrightarrow$ Characteristic Equations
  - $\hookrightarrow$ Undetermined Coefficients

The Trouble with Euler:

$\rightarrow$ You took the slope on the left. $\leftarrow$

Start at the initial time t_0 and initial position y_0.

Choose a stepsize Δt.

- $t_{k+1} = t_k + \Delta t$
- $y_{k+1} = y_k + f(t_k, y_k)\Delta t$

IDEA: Use the "average" slope

Heun's Method

Start at the initial time t_0 and initial position y_0.

Choose a stepsize Δt.

- $t_{k+1} = t_k + \Delta t$
- $\text{Slope}_1 = f(t_k, y_k)$

  $\text{Slope}_2 = f(t_k, y_k + \text{Slope}_1 \Delta t)$
- $y_{k+1} = y_k + \left[\dfrac{\text{Slope}_1 + \text{Slope}_2}{2}\right] \Delta t$

Homework Problem:

Change the procedure in euler.ms to run Heun's method.

Linear Equations So Far...

- General Solution = Homogeneous + Particular Solution
- Find a Homogeneous Solution
  - ↪ Separate and Integrate
  - ↪ Add C.E.
- Find a Particular Solution
  - ↪ Vary Your Parameters
  - ↪ Use the Integrating Factor
  - ↪ Add U.C.

Characteristic Equation

Find the homogeneous solution for

$$ay' + by = f(x)$$

when a and b are <u>constants</u>!

Assume that the solution is $e^{\lambda x}$ and solve for λ:

Plug it in:

$$a\left[e^{\lambda x}\right]' + b\left[e^{\lambda x}\right] = 0$$

Call this the <u>characteristic equation</u>.

Examples:

- $3y' + 5y = e^{-2x}$ .... C.E. $3\lambda + 5 = 0$
- $P' - kP = E$ .... C.E. $\lambda - k = 0$
- $y' + \pi y = x^3 - 2x + 1$ ..... C.E. $\lambda + \pi = 0$

Undetermined Coefficients

For constant coefficient equations with special forcing functions, you can guess the form of the particular solution.

Examples:

- $3y' + 5y = e^{-2x}$ .... y_p "looks like" e^{-2x}
- $3y' + 5y = \sin(2t)$ .... y_p "looks like" $\sin(2t)$
- $3y' + 5y = x^3 - 2x + 1$ .... y_p "looks like" a cubic

U.C. Details

Find a particular solution for

$$3y' + 5y = e^{-2x}$$

Guess

$$y_p(x) = Ae^{-2x}$$

Plug it in and solve for A:

U.C. Details

Find a particular solution for

$$3y' + 5y = \sin(2t)$$

Guess

$$y_p(t) = A\sin(2t) + B\cos(2t)$$

Plug it in and solve for A and B:

One Bad Example

Find a particular solution for

$$y' + 5y = e^{-5x}$$

Guess $y_p(x) = Ae^{-5x}$ and plug it in:

$$\left[Ae^{-5x}\right]' + 5Ae^{-5x} = -5Ae^{-5x} + 5Ae^{-5x} = 0$$

You cannot solve for A!

Why? The forcing term is a homogeneous solution!

HW #4: Due Tomorrow in Conference

Sec 3.3: 6, 11, 12, 24, 32 ;

Sec 3.5: 5, 6, 9 ;

Adapt euler.ms for Heun's method

Do you have a Project Team?

Sign up with PLA!!!

Test #1: Next Wednesday (February 5) in Lecture