TOPICS

- Summary CE and UC
  - $\hookrightarrow$ Constant Coefficients!
  - $\hookrightarrow$ Special Forcing Terms
- Qualitative Behavior
  - $\hookrightarrow$ Graph the solution

    (without knowing the solution)
- Thinking about Test #1

Start with One Bad Example

Find a particular solution for

$$y' + 5y = e^{-5x}$$

Trouble: The forcing term is a homogeneous solution!

Vary your parameters:

$$u(x) = \int \frac{f(x)}{y_h(x)} dx = \int \frac{e^{-5x}}{e^{-5x}} dx = \int 1 dx = x$$

Implication: $y_p(x) = u(x)e^{-5x} = xe^{-5x}$

Check it:

$$\left(e^{-5x} - 5xe^{-5x}\right) + 5xe^{-5x} = e^{-5x}$$

Undetermined Coefficients Works

Make a Table:

- $f(x) = e^{\lambda x}$ .... $y_p(x) = Ae^{\lambda x}$
- $f(x) = \sin(\omega t)$ .... $y_p(x) = A\sin(\omega t) + B\cos(\omega t)$
- $f(x) = x^n$ ..... $y_p(x) = A_0 + A_1 x^1 + A_2 x^2 + \cdots + A_n x^n$

Undetermined Coefficients Fails

Examples:

- $y' - 2y = x^{-2}$
- $3y' + 5y = \tan(\omega t)$
- $xy' + y = 3e^{4x}$

U.C. Details

Find a particular solution for

$$xy' + y = 3e^{4x}$$

Guess

$$y_p(x) = Ae^{4x}$$

Plug it in and solve for A:

$$x4Ae^{4x} + Ae^{4x} = e3e^{4x} \iff (4x+1)A = 3$$

Example: Graph several solution curves for

$$y' = y^2 - 1$$

Keys:

- $y(t)$ is <u>constant</u> when $y'(t) = 0 \iff y^2 - 1 = 0$
  $y = \pm 1$: Call these *steady states.*
- $y(t)$ is increasing when $y_p(t) > 0$

$$\iff y > 1 \text{ or } y < -1$$

- $y(t)$ is decreasing when $y_p(t) < 0$

$$\iff y^2 < 1 \iff -1 < y < 1$$

Aside:

The "solution" for $y' = y^2 - 1$ is

$$y(t) = \tanh(C - t)$$

Would you rather build the graph from this?

Checking Concavity

You need the second derivative.

$$y' = y^2 - 1 \Longrightarrow y'' = 2yy' = 2y(y^2 - 1)$$

Implications:

$\hookrightarrow$ Concave up for $y''(t) > 0$

$$\Longleftrightarrow (y > 1) \text{ or } (-1 < y < 0)$$

$\hookrightarrow$ Concave down for $y''(t) < 0$

$$\Longleftrightarrow (0 < y < 1) \text{ or } (y < -1)$$

Make a Table:

Make a Table:

Use: $y' = y^2 - 1 \qquad y'' = 2yy' = 2y(y^2 - 1)$

Test #1: Guaranteed Problems

1. Solve a linear equation.
   - $\rightarrow$ Find homogeneous, particular, and general solutions.
   - $\rightarrow$ Use initial data.
   - $\rightarrow$ "Analyze" the solution.
2. Work with a model.
   - $\rightarrow$ Build the ODE from scratch.
   - $\rightarrow$ *Explain why* you do what you do.
3. One "challenging" problem.

(Look at the sample tests on the Web)

Sample Problem:

Analyze following differential equation:

$$y' + 4y = 100t,\ y(0) = 200$$

(a) Find a nontrivial solution to the corresponding homogeneous equation.

(b) Find a particular solution to the original equation.

(c) Give the general solution and find the unique solution satisfying the initial condition.

(d) Describe the behavior of the solution for large values of the independent variable t.

Another Sample Problem:

Given that

- $L(t)$ solves $y' + t^3 y = 0$ with $L(0) = 5$,

and

- $M(t)$ solves $y' + t^3 y = \cos(2\pi t)$ with $M(0) = -5$,

find the solution, in terms of L and M, for

$$y' + t^3 y = \cos(2\pi t), \quad y(0) = 10\ .$$

HW #5: Due Monday in Lecture

Sec 3.8: 4, 5, 6, 25;

Sec 4.1: 3, 7, 8

The Worcester Yeast Problem

(Problem 2 on the sample test.)

Test #1: Next Wednesday in Lecture

Closed Book, Closed Notes, Calculator is Okay