MA 2051 B97 Name_____
Exam 1 Section_____
November 18, 1997

There are six pages, including one blank sheet. Show your work in a clear and organized fashion, and include verbal explanations when appropriate. If you need more room, you may write on the back of the pages or on the extra sheet provided; but indicate *clearly* where your solution is continued. Remember, you must show your work to receive full credit.

1. (20 points) Solve each of the following initial-value problems:

   a. $\dfrac{dy}{dx} = x + 2;\ y(2) = 3.$

   b. $\dfrac{dy}{dx} = y + 2;\ y(2) = 3.$

2. (30 points) Consider the differential equation

$$\frac{dx}{dt} = 4x - 5\cos(2t)$$

a. Find a nontrivial solution to the homogeneous equation.

b. Find a particular solution to the differential equation

c. Find the general solution to the equation.

d. Consider the solutions with two different initial values. Do the solutions get farther apart or closer together as t increases? Justify your answer.

3. (30 points) Water is being poured into a tank at a constant rate of 3 liters per second. A small leak at the bottom of the tank causes water to leak out at a rate proportional to the amount of water in the tank. Suppose that initially there is 400 liters of water in the tank and that initially the leaking rate is 4 liters per second.

   a. Write an expression for the amount of water that is added to the tank in a small time period. Write an expression for the amount of water that leaves the tank in the same small time period. Define any variables you use.

   b. Write an equation that gives the change in the amount of water over a small time period. Use this equation to create a differential equation model for the amount of water in the tank as a function of time.

   c. What is the physical meaning of the free and forced response of this model?

4. (25 points) Consider the differential equation $u' = 2 - u + u^2$.

   a. Find the equilibria of this differential equation.

   b. Where are solutions to this equation increasing? Where are they decreasing?

   c. Decide whether each equilibrium in part (a) is stable or unstable.