TOPICS

- You've ALMOST made it!
  - ↪ TEST II: Wednesday at 9:00am
  - ↪ Closed-book, No notes, Calculator OK
- REVIEW PROBLEMS

Announcements:

- Review Session for Test II:
  - ↪ Tuesday, March 2: 7:00pm in SL115
- Regular Help Session
  - ↪ Tonight, 6:3pm in SH304

Guaranteed Problems:

1. Solve a second-order equation
   - ↪ Homogeneous Solution & Particular Solution
   - ↪ Initial Data
2. Analyze a spring-mass sytem
   - ↪ Period, Frequency, Amplitude...
   - ↪ Under, Over, Critical Damping...
   - ↪ Resonance...
3. Solve a system of linear equations
   - ↪ CE and Homogeneous Solution
   - ↪ Find a Particular Solution
   - ↪ Plots in the Phase Plane

Problem #1: Analyze a Linear Equation

$$y'' + 3y' + 2y = 3\cos(2t) - \sin(2t)$$

- Solve the homogeneous equation.
- Find the particular and the general solution.
- Add initial data.
- Find the transient component.
- Is there a limiting periodic solution?

Homogeneous Solution: CE

$$y'' + 3y' + 2y = 0$$

Characteristic Equation: $r^2 + 3r + 2 = (r+2)(r+1) = 0$

Distinct real roots: $y_1(t) = e^{-1t},\ y_2(t) = e^{-2t}$

Test Linear Independence: $W(y_1, y_2) =$???

Undetermined Coefficients:

$$y_p(t) = A\sin(2t) + B\cos(2t)$$

Plug it in:

$$\begin{aligned} 3\cos(2t) - \sin(2t) &= y_p'' + 3y_p' + 2y_p \\ &= (-4A - 6B + 2A)\sin(2t) \\ &\quad +(-4B + 6A + 2B)\cos(2t) \end{aligned}$$

Sine Term: $-2A - 6B = -1$

Cosine Term: $6A - 2B = 3$

$$A = \frac{1}{2},\ B = 0 \implies y_p(t) = \frac{1}{2}\sin(2t)$$

Keep Going...

- General Solution: $y(t) = C_1 e^{-t} + C_2 e^{-2t} + \frac{1}{2}\sin(2t)$
- Find the forced solution: Set $y(0) = 0$, $y'(0) = 0$.

$$\begin{aligned} 0 &= y(0) = C_1 + C_2 + 0 \\ 0 &= y'(0) = -C_1 - 2C_2 + 1 \end{aligned}$$

Solution: $C_1 = -1$ and $C_2 = 1$, so the forced solution is

$$y(t) = -e^{-t} + e^{-2t} + \frac{1}{2}\sin(2t)$$

And Going...

- The transient part of the solution is $-e^{-t} + e^{-2t}$

  Solve $e^{-2t} - e^{-t} = \pm 0.0001$

$$e^{-t} = \frac{1 \pm \sqrt{1 - 0.0004}}{2} \implies t \approx 9.210$$

- There is a limiting periodic solution: $\frac{1}{2}\sin(2t)$.

  The amplitude is $A = 1/2$ and the period is $T = \pi$.

Problem #2: Analyze a Spring-Mass System

A mass of 6kg is suspended from a spring that extended 1cm when a 1kg mass was attached. Pull the mass down 5cm and release it from rest.

- Basic Model: No damping, No forcing:

$$mx''(t) + kx(t) = 0, \quad x(0) = x_i,\ x'(0) = v_i.$$

- Determine some physical parameters...
- Add forcing...
- Add damping...

Analyze a Spring-Mass System

A mass of 6kg is suspended from a spring that extended 1cm when a 1kg mass was attached. Pull the mass down 5cm and release it from rest.

- Fix the parameters:

$$m = 6, \quad k = \frac{1 \cdot 9.8}{0.01} = 980, \quad x_i = -0.05, \quad v_i = 0$$

- Natural frequency $\omega_n = \sqrt{\frac{k}{m}} \approx$ _____

  Period $T = \frac{2\pi}{\omega_n} \approx$ _____

Keep Going ...

- Solve the IVP:

  General Solution $C_1 \sin(\omega_n t) + C_2 \cos(\omega_n t)$

  Use the initial data to get $x(t) = -0.05 \cos(\omega_n t)$

- Amplitude $= 0.05$ meters.

- Make the damping critical:

  $\hookrightarrow p^2 \leq 4mk \implies p \approx$ _____

Question:

What conditions on a, b, c, d guarantee that the homogeneous solution will be transient?

$$\begin{aligned} y' &= ay + bz + f(t) \\ z' &= cy + dz + g(t) \end{aligned}$$

- **Test II: Wednesday, 9:00am, Here**
- **Review Session: 7:00pm in SL115**
- **Project #2:** Due Thursday at Noon in SH202A

  Hand it Wednesday in Lecture: +3 points