TOPICS

- More Numerical Methods
  - $\hookrightarrow$ Slope in the Middle?
- More Linear Equations
  - $\hookrightarrow$ Focus on Constant Coefficients
    - $\hookrightarrow$ Characteristic Equations
    - $\hookrightarrow$ Undetermined Coefficients

The Trouble with Euler:

$\rightarrow$ You took the slope on the left. $\leftarrow$

Start at the initial time t_0 and initial position y_0.

Choose a stepsize Δt.

- $t_{k+1} = t_k + \Delta t$
- $y_{k+1} = y_k + f(t_k, y_k)\Delta t$

IDEA: Use the "average" slope

Recall Heun's Method

Start at the initial time t_0 and initial position y_0.

Choose a stepsize Δt.

- $t_{k+1} = t_k + \Delta t$
- $\text{Slope}_1 = f(t_k, y_k)$

  $\text{Slope}_2 = f(t_k, y_k + \text{Slope}_1 \Delta t)$
- $y_{k+1} = y_k + \left[\dfrac{\text{Slope}_1 + \text{Slope}_2}{2}\right] \Delta t$

One Example:

Linear Equations So Far...

- General Solution = Homogeneous + Particular Solution
- Find a Homogeneous Solution
  - $\hookrightarrow$ Separate and Integrate
  - $\hookrightarrow$ Add C.E.
- Find a Particular Solution
  - $\hookrightarrow$ Vary Your Parameters
  - $\hookrightarrow$ Use the Integrating Factor
  - $\hookrightarrow$ Add U.C.

Characteristic Equation

Find the homogeneous solution for

$$ay' + by = f(x)$$

when a and b are <u>constants</u>!

Assume that the solution is $e^{\lambda x}$ and solve for λ:

Plug it in:

$$a\left[e^{\lambda x}\right]' + b\left[e^{\lambda x}\right] = 0$$

Call this the <u>characteristic equation</u>.

Examples:

- $3y' + 5y = e^{-2x}$ .... C.E. $3\lambda + 5 = 0$
- $P' - kP = E$ .... C.E. $\lambda - k = 0$
- $y' + \pi y = x^3 - 2x + 1$ .... C.E. $\lambda + \pi = 0$

Undetermined Coefficients

For <u>constant coefficient</u> equations with <u>special</u> forcing functions, you can guess the form of the particular solution.

Examples:

- $3y' + 5y = e^{-2x}$ .... y_p "looks like" e^{-2x}
- $3y' + 5y = \sin(2t)$ .... y_p "looks like" $\sin(2t)$
- $3y' + 5y = x^3 - 2x + 1$ .... y_p "looks like" a cubic

U.C. Details

Find a particular solution for

$$3y' + 5y = e^{-2x}$$

Guess

$$y_p(x) = Ae^{-2x}$$

Plug it in and solve for A:

U.C. Details

Find a particular solution for

$$3y' + 5y = \sin(2t)$$

Guess

$$y_p(t) = A\sin(2t) + B\cos(2t)$$

Plug it in and solve for A and B:

One Bad Example

Find a particular solution for

$$y' + 5y = e^{-5x}$$

Guess $y_p(x) = Ae^{-5x}$ and plug it in:

Why? The forcing term is a homogeneous solution!

Back to V of P!

Build a particular solution for

$$y' + 5y = e^{-5x}$$

from

$$y_p(x) = u(x)y_h(x) = u(x)e^{-5x}$$

You will get a nice formula for $u(x)$:

Collect HW #4 Tomorrow

Sec 3.3: 6, 11, 12, 24, 32 ;

Sec 3.5: 5, 6, 9 ;

HW #5: Due Monday in Lecture

Sec 3.8: 4, 5, 6, 25 ;

Sec 4.1: 3, 7, 8 ;

Test #1: Next Wednesday (February 3) in Lecture

Closed Book and Notes

Calculator is Okay