TOPICS

- Summary CE and UC
  - $\hookrightarrow$ Constant Coefficients!
  - $\hookrightarrow$ Special Forcing Terms
- Qualitative Behavior
  - $\hookrightarrow$ Graph the solution...

    without knowing the solution formula!
- Thinking about Test #1

Undetermined Coefficients Works

Make a Table:

- $f(x) = e^{\lambda x}$ .... ____________________
- $f(x) = \sin(\omega t)$ .... ____________________
- $f(x) = \cos(\omega t)$ .... ____________________
- $f(x) = x^n$ .... ____________________

What about sums and products?

Recall One Bad Example (Quiz 2)

Find a particular solution for

$$y' + 5y = e^{-5x}$$

Trouble: The forcing term is a homogeneous solution!

Plug it in:

An Easy Fix

Vary your parameters:

$$u(x) = \int \frac{f(x)}{y_h(x)} dx = \int \frac{e^{-5x}}{e^{-5x}} dx = \int 1 dx = x$$

Implication: $y_p(x) = u(x)e^{-5x} = xe^{-5x}$

UC Says: Guess

$$y_p(x) = Axe^{-5x}$$

This will always work!

Undetermined Coefficients Fails

Examples:

- $y' - 2y = x^{-2}$
- $3y' + 5y = \tan(\omega t)$
- $xy' + y = 3e^{4x}$

Try it anyway...

Find a particular solution for

$$xy' + y = 3e^{4x}$$

Guess

$$y_p(x) = Ae^{4x}$$

Plug it in and solve for A:

Example: Graph several solution curves for

$$y' = y^2 - 1$$

Keys:

- $y(t)$ is <u>constant</u> when $y'(t) = 0 \Longleftrightarrow y^2 - 1 = 0$
  $y = \pm 1$: Call these *steady states.*
- $y(t)$ is increasing when $y'(t) > 0$
- $y(t)$ is decreasing when $y'(t) < 0$

Aside:

The "solution" for $y' = y^2 - 1$ is

$$y(t) = \tanh(C - t)$$

Would you rather build the graph from this?

Checking Concavity

You need the second derivative.

Implications:

$\hookrightarrow$ Concave up for $y''(t) > 0$

$\hookrightarrow$ Concave down for $y''(t) < 0$

Make a Table:

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Use: $y' = y^2 - 1$ $\qquad y'' = 2yy' = 2y(y^2 - 1)$

Test #1: Guaranteed Problems

1. Solve a linear equation.
   - → Find homogeneous, particular, and general solutions.
   - → Use initial data.
   - → "Analyze" the solution.
2. Work with a model.
   - → Build the ODE from scratch.
   - → *Explain why* you do what you do.
3. One "challenging" problem.

(Look at the sample tests on the Web)

Sample Problem:

Analyze following differential equation:

$$y' + 4y = 100t, \; y(0) = 200$$

(a) Find a nontrivial solution to the corresponding homogeneous equation.

(b) Find a particular solution to the original equation.

(c) Give the general solution and find the unique solution satisfying the initial condition.

(d) Describe the behavior of the solution for large values of the independent variable t.

Another Sample Problem:

Given that

- $L(t)$ solves $y' + t^3 y = 0$ with $L(0) = 5$,

and

- $M(t)$ solves $y' + t^3 y = \cos(2\pi t)$ with $M(0) = -5$,

find the solution, in terms of L and M, for

$$y' + t^3 y = \cos(2\pi t), \quad y(0) = 10 .$$

Bonus Problem: Due Monday in Lecture

Explain why *all solutions* for a differential equation of the form

$$\frac{dx}{dt} = f(x)$$

are either monotone increasing, monotone decreasing, or constant. (Oscillations just cannot happen!)

HW #5: Due Monday in Lecture

Sec 3.8: 4, 5, 6, 25;

Sec 4.1: 3, 7, 8

The Worcester Yeast Problem

(Problem 2 on the sample test.)

Test #1: Next Wednesday in Lecture

Closed Book, Closed Notes, Calculator is Okay