MA 2051–C99 **Project #2A**

Nonlinear Shocks

You have been working with models for oscillating systems with damping and external forcing. One application for the model is an automobile suspension system, where the forcing term is the road (with or without frost heaves) and shock absorbers provide the damping.

In this project, you will study the behavior of the solution with different types of damping terms. Notice that once you allow the damping coefficient to depend on the solution, the equation is no longer linear and most analytic solution methods (such as characteristic equations and undetermined coefficients) are useless. You have no choice but to study the system through numerical methods.

1. To start, describe the limiting behavior of the standard model

   $$mx''(t) + px'(t) + kx(t) = F\sin(\omega t), \quad x(0) = x_i,\ x'(0) = v_i\ .$$

   One of the best ways to analyze the solution curves for a second-order equation is to graph them in the $(x, v) =$ (position, velocity) plane. Derive a system of first-order differential equations for $x(t)$ and $v(t) = x'(t)$ and plot solution curves in the (x, v) plane. Pay particular attention to the limiting behavior of the solutions. How does the limit depend on the initial data? How does it depend on other parameters in the model?

2. Mathematicians and engineers at the Renault Automobile Company have studied a new type of shock absorber which controls the damping coefficient. Their new shock absorber can increase the damping coefficient whenever the mass (i.e., the car) is returning toward equilibrium. In the mathematical model, the damping coefficient is a discontinuous function of both $x(t)$ and $v(t)$.

   Do some numerical experiments with the Renault shock absorber. Explain in detail how you program the damping coefficient in your equation. Take $F\sin(\omega t)$ for the forcing function and describe the limiting behavior of the solution for a range of ω values. If you look carefully, you should find that the behavior changes drastically for some special values of ω. Does the new shock absorber give a "smoother" ride than the original shock absorber?

Your Report: Your analysis should include *qualitative, numerical,* and *analytical* work, as well as graphs of data and solutions for your models. Your report will be graded for both technical merit and quality of communication.

- **Final Draft Due: Wednesday, March 3**