MA 2051–C99 **Project #2B**

Scanning for Resonance

If you know all of the parameters in a model, be it a spring-mass system or an RCL circuit, you can analyze the model by building solutions.

In some situations the parameters can change with time. In one particularly important case, it is the frequency of the forcing term that is changing. For example, if the spring mass system is your car, then the frequency of forcing changes with the road and your speed. If the mechanical system is your washing machine, then the forcing frequency changes as the washer enters the spin cycle. You saw one more example in the video on "passage through resonance"

In this project, you will look a little deeper into the behavior of damped and forced oscillators when the parameters are allowed to change.

1. The basic model for the forced system is

   $$mx'' + px' + kx = F\sin(\omega_{\text{drv}} \cdot t)$$

   and you know how to solve this system when ω_{drv}, the driving frequency, is constant.

   It is "standard" for engineers to rewrite this equation as

   $$x'' + 2\zeta\omega x' + \omega^2 x = \hat{F}\sin(\omega_{\text{drv}} \cdot t)$$

   Derive the formulae for ζ and ω and explain how to characterize the system as over, under, or critically damped in terms of the new parameters. What is the resonant frequency in terms of the new parameters? What is the amplitude at resonance for the system?

2. What happens when the driving frequency ω_{drv} changes with time? The model is still valid, but the old solution formulas are not. Verify this statement. Can you give a solution formula for the case $\omega_{\text{drv}}(t) = \omega_0 + \alpha t$, where ω_0 and α are parameters? Plot some solution curves to see how they differ from the solutions for constant ω_{drv}.

   Study how the maximum displacement depends on α when $\omega_{\text{drv}} = \omega_0 + \alpha t$. How does the maximum displacement compare with the actual amplitude at resonance (for the case when the forcing frequency is constant)? Can the response amplitude be reduced by passing through resonance quickly?

3. In one kind of mechanical shaker test, a system is subjected to periodic loading with an increasing (or decreasing frequency). These tests, which are used to scan for the resonant frequency, are sometimes referred to as "chirping."

   Small α would correspond to a slow scan for resonance. Large α would correspond to a fast scan. Find an appropriate scanning speed (α) which would be adequate to estimate the steady-state resonance amplitude. How do you measure the adequacy of your speed?

Your Report: Your analysis should include *qualitative, numerical,* and *analytical* work, as well as graphs of data and solutions for your models. Your report will be graded for both technical merit and quality of communication.

- **Final Draft Due: Wednesday, March 3**