**MA 2051–C99** **Project #2C**

## Mathematical Epidemics

There has been increased interest recently for mathematical models of the spread of contagious disease through a population. The purpose of this project is to explore one class of models along with some implications for health-care policy.

One mathematical model for the spread of an infectious disease divides the population into *Susceptible*, *Infective*, and *Removed* classes. A susceptible individual can catch the disease by contact with an infected individual. An infected individual moves on to the removed class by being isolated from the population, recovering (and developing immunity) or dying. Some of the recovered class may eventually loose immunity and return to the susceptible class. Here is a model to start with:

$$\begin{aligned}\frac{dS}{dt} &= -\beta IS + \gamma R \\ \frac{dI}{dt} &= \beta IS - \nu I \\ \frac{dR}{dt} &= \nu I - \gamma R\end{aligned}$$

1. Translate the model into English: what does each term in the equation represent? Show that the total number $N = S + I + R$ remains constant in this model. Use this fact to reduce the model to a system of two differential equations (for $S$ and $I$).

2. Work with the special case $\gamma = 0$. In this case, once an individual is removed, he or she never becomes susceptible again. Determine conditions (on $\beta$, $\nu$, $N$ and the initial data) under which $I(t)$ will increase initially. What happens for large $t$? Does the disease spread through the entire population?

   You may use your favorite software (Matlab or Maple or...) to investigate the system and obtain solution curves.

3. Assume now that $\gamma > 0$. Again, study the long-term behavior of the solutions $S(t)$ and $I(t)$. The answer depends on the parameters $\beta$, $\gamma$, $\nu$, and $N$.

   Remark: in some literature on this subject, the quantity $R_0 = N(\beta/\nu)$ is called the ***intrinsic reproductive rate of the disease***. For a recent smallpox epidemic in developing countries, $R_0$ was estimated to be approximately 4. For a measles outbreak in the U.S. in the early part of this century, $R_0 \approx 13$.

4. One important application for this model considers vaccination of the susceptibles (to give them immunity). Describe how to include vaccinations in this model. How much of the population must be vaccinated in order to control the disease?

**Your Report:** Your analysis should include *qualitative, numerical,* and *analytical* work, as well as graphs of data and solutions for your models. Your report will be graded for both technical merit and quality of communication.

- **Final Draft Due: Wednesday, March 3**