1. Suppose that $A$ is an invertible matrix that satisfies $A^{-1} = A^T$. Use the properties of the determinant to show that $\det(A) = \pm 1$. Make sure you note which properties you are using.

2. Show that the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

is invertible, and compute its inverse. You may use either the special formula for a $2 \times 2$ matrix, or the general method involving RREF.

3. Suppose $A$ and $B$ are $n \times n$ matrices and that $A$ is invertible but $AB$ is not invertible. What can you say about the determinant of $B$? What can you conclude about the columns of $B$? Justify your answers.

4. Let $V$ be the set of all ordered pairs, $(x, y)$. Define the · operation by $c \cdot (x, y) = (cx, -cy)$. Show that this definition does not satisfy axiom (10) given below.

Axiom (10) $1 \cdot u = u$.

5. Suppose that $W$ is the subset of $\mathbb{R}^4$ given by all vectors of the form $(a, b, c, d)$ with $a - c = 0$ and $b + c + d = 0$. Show that $W$ is a subspace of $\mathbb{R}^4$.

6. Let $W$ be the subset of $\mathbb{P}_2$ consisting of all polynomials of the form $a + bt + ct^2$ with $a + b = 0$. Show that $W$ is a subspace of $\mathbb{P}_2$.

7. Suppose that $A$ is an $m \times n$ matrix. Explain what it means for a vector $b$ to be in the column space of $A$.

8. Let $V$ be the vector space of all functions that are continuous on the interval $[0, 1]$. Let $T : V \rightarrow \mathbb{R}$ be defined by

$$T(f) = \int_0^1 f(x) \, dx$$

Show that $T$ is a linear transformation. Then explain why $T$ cannot be one to one.

9. Find a set of vectors that spans the null space of the matrix given below.

$$A = \begin{bmatrix} 1 & 2 & -4 & 3 \\ 0 & 1 & -2 & 0 \\ 2 & -1 & 2 & 6 \\ -1 & 3 & -6 & -3 \end{bmatrix}$$