1. Determine if the following set of polynomials spans $P_2$.
\[ \{t^2 + 2, t^2 - 2, t + 1\} \]

2. Let $W$ be the set of all vectors in $\mathbb{R}^4$ of the form shown below, where $a$, $b$, and $c$ represent arbitrary real numbers. Either find a set of vectors that spans $W$ or give an example that shows that $W$ is not a vector space.
\[
\begin{pmatrix}
  a + b + c \\
  2a - b \\
  0 \\
  3a - 2b - 5c
\end{pmatrix}
\]

3. Consider the following matrix.
\[
\begin{pmatrix}
  1 & 0 & 0 \\
  1 & 2 & 0 \\
  3 & 4 & 4
\end{pmatrix}
\]
First, compute the determinant of this matrix. If the matrix is invertible, compute its inverse.

4. Let $V$ be the set of all ordered pairs of real numbers. Define the addition operation by $(x_1, y_1) + (x_2, y_2) = (x_2, y_1 + y_2)$. Show that this definition does not satisfy axiom (4) (given below) of a vector space.
Axiom (4) There is a zero vector $\mathbf{0}$ in $V$ such that $u + \mathbf{0} = u$ for all $u$ in $V$.

5. Let $A$ be an $n \times n$ matrix. Suppose that the linear transformation given by $\mathbf{x} \rightarrow A\mathbf{x}$ is not one-to-one. List five other distinct statements about $A$ that must be true.

6. Let $T : \mathbb{P}_3 \rightarrow \mathbb{P}_2$ be defined by $T(p) = p'(t)$. That is, $T$ acts by differentiating. Show that $T$ is linear and find the kernel of $T$. Is $T$ one-to-one? Is $T$ onto? Justify your answers.

7. Suppose $A$ and $B$ are $n \times n$ matrices and that $A$ is invertible but $AB$ is not invertible. What can you say about the determinant of $B$? What can you conclude about the columns of $B$? Justify your answers.

8. Let $V$ be the vector space of all functions that are continuous on the interval $[0, 1]$. Let $T : V \rightarrow \mathbb{R}$ be defined by
\[
T(f) = \int_0^1 f(x) \, dx
\]
Show that $T$ is a linear transformation. Then explain why $T$ cannot be one to one.