**MA 2071 Linear Algebra** **Name:** ______________________

**Final a, E Term 2013** **Section:** ______________________

Show all work needed to reach your answers.

1. (20 points)

   (a) Please compute the determinant of the following matrix: $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 2 & -1 & 3 \end{bmatrix}$

   determinant value: __________

   (b) Is the above matrix singular or nonsingular? If it is singular, please explain why; if it is nonsingular, please compute its inverse.

2. (6 points) Think carefully; compute little.

$$\begin{bmatrix} 1 & -1 & 2 & 6 & -1 \\ 0 & 2 & 1 & 5 & 4 \\ 0 & 0 & -1 & -2 & 7 \\ 0 & 0 & 0 & 4 & 11 \\ 0 & 0 & 0 & 0 & -3 \end{bmatrix}$$

(a) What are the eigenvalues of the matrix?

(b) Why or why not is the above matrix diagonalizable?

(c) What is the determinant of this matrix?

3. (14 points) Please find a matrix $A$ with eigenvector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ corresponding to the eigenvalue 3 and eigenvector $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ corresponding to the eigenvalue 5.

$A =$ ________________

4. (20 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & -4 & 3 \\ 0 & 1 & -2 & 0 \\ 2 & -1 & 2 & 6 \\ -1 & 3 & -6 & -3 \end{bmatrix}$$

(a) Please find a basis, $\mathfrak{B}_1$, for $\mathrm{Col}(A)$, the column space of $A$.

(b) Please find a basis, $\mathfrak{B}_2$, for $\mathrm{Null}(A)$, the null space of $A$.

5. (10 points) In terms of least squares, which line best fits that points $(1, -1)$, $(2, 4)$, and $(0, -3)$?

6. (10 points) Are these vectors is linearly dependent or linearly independent? Please justify your answer.

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} \right\}$$

7. (12 points) Please use the Gram-Schmidt process to find an orthonormal basis for $\mathbb{R}^3$ using in order the basis vectors

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \right\}$$

8. (8 points) Suppose $\mathcal{V}$ is a vector space over the reals. Suppose also that $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are both vectors in $\mathcal{V}$, and $\alpha \in \mathbb{R}$.

   (a) According to the definition of vector space,

   $$\alpha(\boldsymbol{v}_1 + \boldsymbol{v}_2) = \underline{\hspace{6cm}}$$

   (b) Which of the following is **not** required by the definition of vector space:

      i. $\boldsymbol{v}_1 + \boldsymbol{v}_2 = \boldsymbol{v}_2 + \boldsymbol{v}_1$

      ii. $\boldsymbol{v}_2 - \boldsymbol{v}_2 = \boldsymbol{0}$

      iii. $\boldsymbol{v}_1 \cdot \boldsymbol{v}_2 = 0$ if and only if $\boldsymbol{v}_1 \perp \boldsymbol{v}_2$

      iv. $1\boldsymbol{v}_1 = \boldsymbol{v}_1$

   Please explain your answer.