MA 2071 Linear Algebra **Name:** ______________________________

Final, E Term 2014 **Section:** ______________________________

Show all work needed to reach your answers. There is 1 free point on this exam.

1. (10 points) Please compute the following determinant: $\begin{vmatrix} 3 & -1 & 2 \\ -2 & 0 & 1 \\ 2 & -1 & 5 \end{vmatrix}$

determinant value: __________

2. (15 points) Please solve the following linear system. Please state whether the system is consistent or inconsistent. Is the solution unique?

$$\begin{array}{rcrcrcrcr} 2x_1 & + & x_2 & + & x_3 & + & 2x_4 & = & 1 \\ x_1 & - & 2x_2 & - & x_3 & & & = & -1 \\ -x_1 & & & + & x_3 & - & 2x_4 & = & 7 \end{array}$$

$x_1 =$ ______________, $x_2 =$ ______________, $x_3 =$ ______________, $x_4 =$ ______________

3. (12 points) For the matrix A below, please find the LU decomposition.

$$A = \begin{bmatrix} 1 & -2 & -1 \\ -1 & 3 & 2 \\ 2 & 1 & 4 \end{bmatrix}$$

$L =$ ____________________ $U =$ ____________________

4. (12 points) Are these vectors linearly dependent or linearly independent? Please justify your answer.

$$\left\{ \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix} \right\}$$

5. (20 points) If $A = \begin{bmatrix} 3 & 9 & 2 & 4 \\ -1 & -3 & 1 & -3 \\ 1 & 3 & 1 & 1 \\ 2 & 6 & 4 & 0 \end{bmatrix}$, please find an orthonormal basis for Null(A).

Orthonormal Basis: ____________________________

6. (5 points) What is the relationship between Null(A) and Row(A)?

7. (15 points) Please find the least-squares solution $\hat{\mathbf{x}}$ to the system $\begin{bmatrix} 2 & 0 \\ 4 & 9 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ -7 \end{bmatrix}$

$\hat{\mathbf{x}} =$ ______

8. (10 points) Suppose a 3×3 matrix A has 2 as a triple eigenvalue (an eigenvalue of multiplicity three) with three linearly independent eigenvectors. Please describe A as complete as possible and justify your answer.