MA 2071 Linear Algebra Name:

Section:

Final 1, E Term 2015

Show all work needed to reach your answers.

1. (10 points) Please find the eigenvalues (not the eigenvectors) of the matrix $A = \begin{bmatrix} 2 & 5 \\ 2 & -1 \end{bmatrix}$

Eigenvalues:

2. (15 points) Please find the inverse of the matrix
$$A = \begin{bmatrix} 1 & -2 & 1 \\ 3 & -5 & 1 \\ -9 & 16 & -4 \end{bmatrix}$$

$$A^{-1} =$$

3. (10 points) Are these vectors is linearly dependent or linearly independent? Please justify your answer.

$$\left\{ \begin{bmatrix} -3\\2\\5 \end{bmatrix}, \begin{bmatrix} 4\\-6\\2 \end{bmatrix}, \begin{bmatrix} -1\\4\\-7 \end{bmatrix} \right\}$$

4. (9 points) If A is a matrix, what are the effects of each of the elementary row operations on det(A)?

1.	
2.	
3.	

5. (11 points) For the matrix A below, please find a basis for the column space of A, Col(A).

	1	-2	1	1
A =	-1	3	0	2
	2	-3	3	-1

Basis:

6. (10 points) Suppose A is a **nonsingular** $n \times n$ matrix which by definition means that A is invertible. Please write five distinct statements about A that are equivalent to **nonsingular**. For example, what can you say about the eigenvalues of A? What can you say about the columns of A?

-

7. (10 points) Suppose that multiplying a 3×3 matrix A by a vector \mathbf{v}_1 doubles the vector, while multiplying A by a vector \mathbf{v}_2 halves the vector, and multiplying A by a vector \mathbf{v}_3 yields the zero vector. What can be said about A?

8. (10 points) Consider the matrix A

$$A = \begin{bmatrix} 0 & -5 \\ 1 & 3 \\ -2 & -1 \end{bmatrix} \quad \text{and the vector} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

For this A and **b**, the system $A\mathbf{x} = \mathbf{b}$ is inconsistent. Please find the best least squares approximate solution, $\hat{\mathbf{x}}$.

9. (10 points) Let $V = \text{Span}\{1, \sin^2(x), \cos^2(x), 2x, x^2\}$. Please give a basis B for V. What two defining conditions must a basis satisfy?

	Basis: $B =$		
Condition 1.			
Condition 2.			
10. (5 points) Suppos	se 2/3 is an eigenvalue of a 3×3 matrix B and that	$\begin{bmatrix} 8\\ -2\\ 1 \end{bmatrix}$	is a corresponding

eigenvector. Please find an eigenvalue and eigenvector for B^2 . Please explain your answer.