MA 2071 Linear Algebra **Name:** ____________________

Final a, E Term 2016 **Section:** ____________________

Show all work needed to reach your answers.

1. (15 points) If possible, please solve the following linear system. Please state whether the system is consistent or inconsistent. Is the solution unique?

$$\begin{array}{rcrcrcr} 3x & - & 2y & - & z & = & -1 \\ x & + & y & & & = & 4 \\ 2x & - & y & - & 3z & = & 5 \end{array}$$

$x =$ ______ $y =$ ______ $z =$ ______

2. (15 points) Are these vectors linearly dependent or linearly independent? Please justify your answer.

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -4 \end{bmatrix} \right\}$$

Please Circle Your Answer: Linearly Independent Linearly Dependent

3. (15 points) Please compute the inverse of the matrix

$$A = \begin{bmatrix} 4 & -1 & 1 \\ 2 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$A^{-1} =$ ________________

4. (15 points) Please find the eigenvalues and corresponding eigenvectors for the matrix
$$\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

First Eigenvalue ________ Corresponding Eigenvector: ________________

Second Eigenvalue ________ Corresponding Eigenvector: ________________

5. (15 points) Think carefully; compute little: please find the value of following determinant:

$$|A| = \begin{vmatrix} 2 & -4 & 2 & 6 \\ 0 & -3 & 9 & 6 \\ 0 & 0 & -1 & 7 \\ 1 & 0 & 0 & 1 \end{vmatrix}$$

$|A| = \det(A) =$ ________________

6. (15 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & -4 \\ 3 & 6 & -1 & -10 \\ -1 & -2 & 3 & -2 \end{bmatrix}$$

Please find an orthogonal basis for the null space of A.

Orthogonal Basis: ______________________

7. (10 points) Can one construct a 3×3 matrix A with eigenvalue $\lambda_1 = 5$ and corresponding eigenvector $\mathbf{v}_1$, eigenvalue $\lambda_2 = 3$ and corresponding eigenvector $\mathbf{v}_2$, and finally eigenvalue $\lambda_3 = 7$ and corresponding eigenvector $\mathbf{v}_3 = \mathbf{v}_1 + \mathbf{v}_2$? Please explain either how to compute A, or why such a matrix can not exist.