MA 2071 Linear Algebra **Name:** ______________________

Final 1, E Term 2017 **Section:** ______________________

Show all work needed to reach your answers.

1. (8 points) For which value of h are these vectors orthogonal?

$$\begin{bmatrix} -3 \\ h \\ -7 \\ 4 \end{bmatrix}, \begin{bmatrix} -5 \\ 2 \\ -1 \\ -3 \end{bmatrix}$$

$h =$ ________

2. (12 points) Please solve the following linear system. Does a solution exist, and is it unique? Please state whether the system is consistent or inconsistent, and find all possible solutions.

$$\begin{array}{rcrcrcr} -x & + & 3y & + & 4z & = & -7 \\ 2x & - & 3y & - & 2z & = & 8 \\ -x & - & 3y & - & 8z & = & 5 \end{array}$$

Solution: ______________________

3. (10 points) Please evaluate this determinant without computing too much: $\begin{vmatrix} 3 & 1 & -2 & -3 \\ -2 & 0 & 4 & 0 \\ 1 & -1 & 0 & -3 \\ 0 & -1 & 0 & -3 \end{vmatrix}$

Determinant Value: ____________

4. (10 points) For the matrix A below, please find a basis for the null space of A, Null(A).

$$A = \begin{bmatrix} 1 & -2 & 1 & 1 \\ -1 & 3 & 0 & 2 \\ 2 & -3 & 3 & -1 \end{bmatrix}$$

Basis: ________________________________

5. (12 points) Consider the matrix $A = \begin{bmatrix} 1 & 3 \\ 0 & -5 \\ -2 & -1 \end{bmatrix}$ and the vector $\mathbf{b} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$

For this A and $\mathbf{b}$, the system $A\mathbf{x} = \mathbf{b}$ is inconsistent. Please find the best least squares approximate solution, $\hat{\mathbf{x}}$.

$\hat{\mathbf{x}} =$ __________

6. (12 points) Please find the 2×2 matrix with eigenvalues $\lambda_1 = 3$, $\lambda_2 = -1$ and eigenvectors $\mathbf{x}_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

Matrix: __________

7. (10 points) Please write the vector $\begin{bmatrix} 1 \\ 9 \\ 2 \end{bmatrix}$ in terms of the basis $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \right\}$

Vector: ____________

8. (10 points) For the matrix A below, please find the LU decomposition.

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ -4 & -6 & 1 & -4 & 1 \\ 0 & 4 & -2 & 3 & -2 \\ 3 & 1 & 1 & 18 & -3 \end{bmatrix}$$

$L =$ ____________________ $U =$ ____________________

9. (10 points) Fill in the blanks:

   - If A is an invertible matrix, then $\det(A^{-1}) =$ ____________________.

   - If a matrix is invertible, then its eigenvalues are ____________________.

   - For any square matrix, the eigenvectors associated with distinct eigenvalues must be ____.

   - For a matrix A with n columns and m linearly independent rows $(m < n)$, then $n - m$ is the dimension of ____________________.

   - For an $m \times n$ matrix, the largest possible number of pivots is ______________.

10. (6 points) Suppose that $A \in \mathcal{M}_{3,3}$ and that A^5 is the zero matrix. Explain why zero is the only possible eigenvalue of A.