MA2431 **Name:** _____

Final Exam **B Term, 2014**

Show all work needed to reach your answers. You may use any theorem we discussed in class, but cite any theorem you use.

1. (15 points) Consider the two ODE systems (a) and (b) below. Which system is a Hamiltonian system? (One is, one is not.) Please find a Hamiltonian for that system.

$$\text{(a)} \quad \begin{aligned} \dot{x} &= y\sin(xy) \\ \dot{y} &= -x\sin(xy) \end{aligned} \qquad\qquad \text{(b)} \quad \begin{aligned} \dot{x} &= x^2 - 2xy \\ \dot{y} &= y^2 - 2xy \end{aligned}$$

Hamiltonian: _____

2. (15 points) Consider a flowing fluid with fluid velocity $\mathbf{v}$. Please define/describe each of the following:

   (a) irrotational flow

   (b) incompressible flow

   (c) total or material derivative $\dfrac{d}{dt}$ in terms of the partial derivative $\dfrac{\partial}{\partial t}$

3. (20 points) The mass of a small rocket just before liftoff is $m = m_0$ grams. After liftoff, because the rocket burns fuel, the mass decreases at a constant rate of $r > 0$ grams per second. The forces that act on the rocket as it rises are gravity, air resistance and the thrust from its engines. Assume that the engine thrust $T > 0$ is constant, that air resistance is proportional to velocity v, and that the rocket heads straight up (one dimensional motion). Please write a first order ODE for v which describes the flight of the rocket while it is burning fuel.

First order ODE: _____

4. (20 points) Consider the system of two ODEs

$$\begin{aligned} \dot{x} &= 4x + y - x(x^2 + y^2) \\ \dot{y} &= -x + 4y - y(x^2 + y^2) \end{aligned}$$

(a) Please find the **radial** equation for the polar coordinate version of this system. Hint: Notice that since $r^2 = x^2 + y^2$, $r\dot{r} = x\dot{x} + y\dot{y}$

(b) What is a limit cycle for this system, and is it stable or unstable? Please explain your answer.

5. (30 points) Consider the first-order system

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= x - x^3 \end{aligned}$$

(a) Please find all equilibrium points for this system.

(b) Please linearize this about $(0, 0)$ and determine the nature (type) of this equilibrium point.

(c) Please find an ODE for dy/dx for curves in the phase plane.

(d) Please separate variables and find the equation for the homoclinic orbits associated with $(0, 0)$.

(e) Please draw a sketch of the phase plane including the equilibrium points and the homoclinic orbit.