MA2431 **Name:** Solutions

Quiz 1 **B Term, 2014**

Show all work needed to reach your answers.

1. (~~10~~ 13 points) The system $\begin{aligned} \dot{x} &= -x^3 - 4y^3 \\ \dot{y} &= 3x^2y + 2 \end{aligned}$ is conservative. Please find a Hamiltonian.

$P(x,y) = \frac{\partial H}{\partial y} \overset{+1}{=} -x^3 - 4y^3 \Rightarrow H(x,y) \overset{+1}{=} \int \frac{\partial H}{\partial y}\, dy \overset{+1}{=}$

$\int (-x^3 - 4y^3)\, dy \overset{+2}{=} -x^3y - y^4 + C(x)$. So

$-Q(x,y) = \frac{\partial H}{\partial x} \overset{+1}{=} -3x^2y - 2 \overset{+2}{=} -3x^2y + C'(x) \Rightarrow C'(x) \overset{+1}{=} -2$

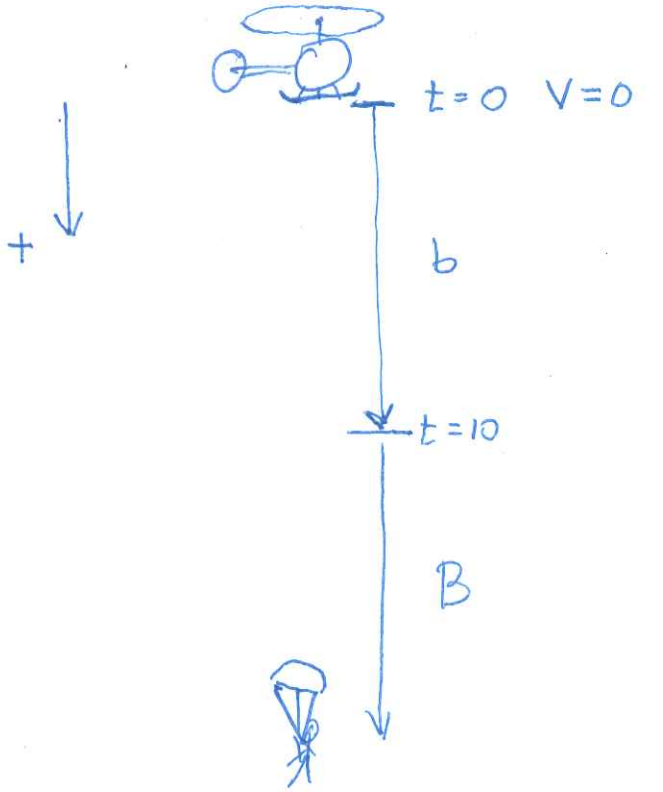
$\Rightarrow C(x) \overset{+1}{=} -2x$. Thus $\boxed{H(x,y) \overset{+1}{=} -x^3y - y^4 - 2x.}$

$+$ -1

12

2. (~~15~~ points) A skydiver drops vertically from rest from a helicopter which is stationary at a height h above the ground. Initially the air resistance force on the skydiver is $F_{drag} = -bv^2$ where $b > 0$ and v is the velocity of the skydiver. After 10 seconds, the skydiver pulls the rip cord, deploying a parachute and increasing the air resistance to $F_{drag} = -Bv^2$ where $B > b$. Please set up ordinary differential equations (ODE) with v as the dependent variable and t (time) as the independent variable for the periods before and after the parachute opens. What are the initial conditions for v at the start of each of these periods?

Do Not Solve This System.



Initial Condition on v when the skydiver leaves the helicopter: (+1) $V(0) = 0$

ODE before parachute opens: (+5) $m\dot{v} = mg - bv^2$

ODE after parachute opens: (+5) $m\dot{v} = mg - Bv^2$

Initial Condition on v when the skydiver pulls the rip cord: (+1) $V(10-) = V(10+)$

meaning that V is continuous between the two periods.