

Show all work needed to reach your answers.

- 5 1. (5 points) Suppose a linear 2×2 system $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = A \begin{bmatrix} x \\ y \end{bmatrix}$ has a matrix A with two real eigenvalues, with $\lambda_- < 0 < \lambda_+$. What type of equilibrium point is $(0,0)$?

There were 3 versions of this question.

saddle point

3.11
iv

2. (10 points) Consider the system $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -1 & -5 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ What type of equilibrium is $(0,0)$ for this system? Justify your answer by computing the appropriate eigenvalues.

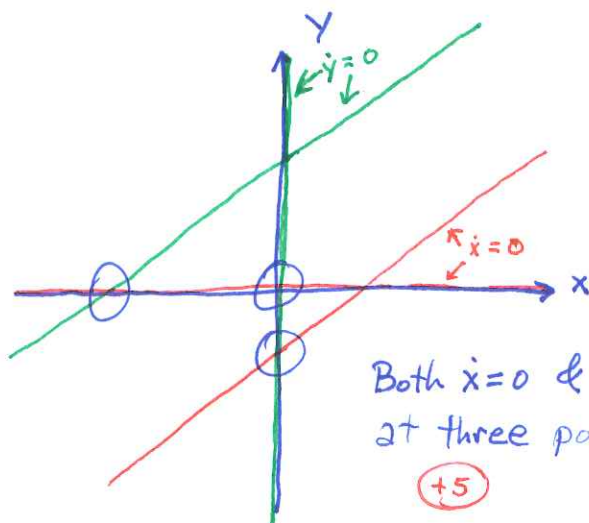
Find eVs:

$$\begin{aligned} \det(A - \lambda I) &= \begin{vmatrix} -1-\lambda & -5 \\ 2 & 5-\lambda \end{vmatrix} = (\lambda-5)(\lambda+1) + 10 = 0 \\ &= \lambda^2 - 4\lambda - 5 + 10 = \lambda^2 - 4\lambda + 5 = 0 \\ &\Rightarrow \lambda_{\pm} = \frac{4 \pm \sqrt{16-20}}{2} = 2 \pm i \end{aligned}$$

Complex eVs with $p=2 > 0$ $\Rightarrow$ This equilibrium point is an unstable spiral

3. (10 points) Please find all of the equilibrium points for the system

$$\begin{aligned} \dot{x} &= y(1-x+y) \\ \dot{y} &= x(2+x-y) \end{aligned} \Rightarrow \begin{cases} y=0 \text{ or } y=x-1 \\ x=0 \text{ or } y=x+2 \end{cases}$$



Both $\dot{x}=0$ & $\dot{y}=0$

at three points: $(0,0), (-2,0), (0,-1)$

(+5)