MA2431 Name: Solutions

Quiz 5 B Term, 2014

Show all work needed to reach your answers.

1. (10 points) Consider the highly modified KdV equation

$$\frac{\partial \psi}{\partial t} + \psi^2 \left(\frac{\partial \psi}{\partial x}\right)^2 + \psi \frac{\partial^3 \psi}{\partial x^3} = 0$$

Please find the 3nd-order KdV ODE for the soliton solution where $\psi(x,t) = f(z)$ and $z := x - vt$ for some constant velocity v.

$$\left.\begin{aligned} \frac{\partial}{\partial t} &= \frac{\partial z}{\partial t}\frac{d}{dz} = -v\frac{d}{dz} \\ \frac{\partial}{\partial x} &= \frac{\partial z}{\partial x}\frac{d}{dz} = 1\frac{d}{dz} \end{aligned}\right\} \Rightarrow -v\frac{df}{dz} + f^2\left(\frac{df}{dz}\right)^2 + f\frac{d^3 f}{dz^3} = 0$$

+5 +5

$$f\frac{d^3 f}{dz^3} + \left(f^2\left(\frac{df}{dz}\right) - v\right)\frac{df}{dz} = 0$$

#4-1 p. 123

2. (15 points) For the modified KdV ODE system,

$$\frac{df}{dz} = y \qquad \frac{dy}{dz} = vf - \frac{f^3}{3}$$

notice that $(0,0)$ is an equilibrium point. Please determine the nature (type) of this equilibrium point by finding the eigenvalues and eigenvectors of the appropriate linearized system.

Linearize about $(0,0)$:

$$\begin{bmatrix} \frac{df}{dz} \\ \frac{dy}{dz} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ v & 0 \end{bmatrix}}_{A} \begin{bmatrix} f \\ y \end{bmatrix}$$ +5

eVs: $\det(A - \lambda I) = \begin{vmatrix} -\lambda & 1 \\ v & -\lambda \end{vmatrix} = \lambda^2 - v = 0$ +5

$\Rightarrow \lambda_{\pm} = \pm\sqrt{v} \in \mathbb{R}$

Since one eV is negative and the other is positive, (0,0) is a saddle point. +2

$e\vec{v}$ corresponding to $\lambda_- = -\sqrt{v}$

$$A - \lambda_- I = \begin{bmatrix} \sqrt{v} & 1 \\ v & \sqrt{v} \end{bmatrix} \Rightarrow \vec{v}_- = \begin{bmatrix} -1 \\ \sqrt{v} \end{bmatrix}$$

$e\vec{v}$ corresponding $\lambda_+ = \sqrt{v}$

$$A - \lambda_+ I = \begin{bmatrix} -\sqrt{v} & 1 \\ v & -\sqrt{v} \end{bmatrix} \Rightarrow \vec{v}_+ = \begin{bmatrix} 1 \\ \sqrt{v} \end{bmatrix}$$

+3