SUPPLEMENTARY EXERCISE SOLUTIONS, CHAPTER 4

AN INTRODUCTION TO STATISTICAL MODELING

S4.1. Researchers conjecture that dyslexia (an information-processing impediment) and handedness are related. To test their conjecture, they randomly select the files of 10,000 elementary school children. Of these, 987 are left handed and 9013 right handed, while 543 are categorized as dyslexic. Of the 543 dyslexic children, 274 are left handed.

a. Suppose L denotes the event that a randomly selected elementary school child is left handed. Based on the given data, what would you estimate the probability of L, $P(L)$, to be? Be sure to justify your answer.

ANS: Since $P(L)$ is a long term relative frequency, we estimate it using the frequency of left handedness in the sample. **(5 points)** This gives the estimate $P_{10000}(L) = 987/10000 = 0.0987$.**(5 points)**

b. If M is the event a randomly selected elementary school child is both left handed and dyslexic, estimate the probability of M.

ANS: (5 points) Reasoning as in part a, $P_{10000}(M) = 274/10000 = 0.0274$.

c. If K is the event a randomly selected elementary school child is neither left handed nor dyslexic, estimate the probability of K.

ANS: (5 points) There are 8744 children who are neither left handed nor dyslexic. Therefore, reasoning as in part a, $P_{10000}(K) = 8744/10000 = 0.8744$.

S4.2. The random variable Y has density

$$\begin{aligned} p_Y(y) &= c/y^3,\ y > 2 \\ &= 0,\ \text{otherwise}. \end{aligned}$$

a. What is the value of c?

ANS:

$$\begin{aligned} 1 &= \int_{-\infty}^{\infty} p_Y(y)dy \ \textbf{(5 points)} \\ &= \int_{2}^{\infty} \frac{c}{y^3}dy \\ &= \left[-\frac{c}{2y^2}\right]_2^{\infty} \\ &= \frac{c}{8} \end{aligned}$$

Therefore, $c = 8$. **(5 points)**

b. What proportion of the population modeled by this probability density function takes values below 3?

ANS: The proportion is

$$\int_2^3 \frac{8}{y^3}dy \ \textbf{(3 points)} = \left[-\frac{8}{2y^2}\right]_2^3 = \frac{5}{9} \ \textbf{(2 points)}$$

c. How relatively likely (e.g., twice as likely) is it that Y occurs in an interval about $y = 4$ of width dy compared with occurring in an interval about $y = 8$ of width dy, where dy is infinitesimally small?

ANS: It is $(8/4^3)/(8/8^3)$ **(3 points)** $= 8$ **(2 points)** times as likely.

S4.4. It seems as if many, if not most, retail prices end in a 9 (e.g., $29.99). To see if this is true, students in a business course took a random sample of 100 retail prices, and found that 19 ended in a 9. Is this convincing evidence that more retail prices end in a 9 than might be expected by chance? Justify your answer using a probability argument. (Hint: Assume a 9 is no more likely to occur than any other integer. Then compute the approximate probability of obtaining 19 or more 9s in a random sample of 100 retail prices.)

ANS: *If retail prices ending in a 9 are no more prevalent than those ending in other digits, then the population proportion of such prices is* $p = 0.1$. **(5 points)** *If* Y *denotes the number of prices in the sample ending in a 9, then* $Y \sim b(100, 0.1)$, **(5 points)** *and the probability of the students observing 19 or more such prices is*

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\begin{aligned}
P(Y \geq 19) &= P(Y \geq 18.5) \text{ (Continuity correction)} \\
&= P\left(\frac{Y - 10}{\sqrt{(100)(0.1)(1-0.1)}} \geq \frac{18.5 - 10}{\sqrt{(100)(0.1)(1-0.1)}}\right) \\
&\approx P(Z \geq 2.83) \text{ (CLT)} \\
&= 0.0023 \text{ (5points)}
\end{aligned}
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This small number constitutes convincing evidence that more retail prices end in a 9 than might be expected by chance. **(5 points)**

S4.9. In the game of ski-ball, players roll a ball up a ramp and onto a disk of radius 5 feet. The disk is divided into 5 sections by raised concentric circles of radii 1, 2, 3, 4, and 5 feet. The player gets 50 points for getting the ball in the smallest circle, 40 points for getting it in the sector between the smallest and second smallest circle, and 30, 20, and 10 points for getting it in the next three sectors. Assume the ball falls on the 5 foot disk at random (meaning there is no skill involved, which is exactly how I play!).

a. Obtain the distribution of Y, the number of points earned on a single roll.

ANS: *The probability of the ball landing in any sector is proportional to the area of the sector. These areas are (smallest to largest):* π, 3π, 5π, 7π, *and* 9π. *Therefore, the distribution of* Y *is* **(10 points)**

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\begin{aligned}
p_Y(y) &= 9/25, \; y = 10 \\
&= 7/25, \; y = 20 \\
&= 5/25, \; y = 30 \\
&= 3/25, \; y = 40 \\
&= 1/25, \; y = 50
\end{aligned}
$$

b. What is the probability of getting at least 30 points on a single roll?

ANS: 9/25 **(5 points)**