1. **(10 points)** Based on historical data, Egbert's travel time from home to work, in minutes, is well modeled by a normal distribution with mean 19 and variance 9. If Egbert leaves home thirteen minutes ahead of an appointment at work, what is the probability he is late for the appointment?

   **ANS:** *Let $Y$ be his travel time. We know $Y \sim N(19, 9)$. The probability he is late is*

$$P(Y > 10) = P\left(\frac{Y-19}{3} > \frac{13-19}{3}\right) = P(Z > -2) = 0.9772$$

2. Market researchers want to estimate the proportion of the population that will respond to an advertizing flyer. To do so, they obtain a random sample of size 50 and find that 5 respond to the flyer.

   (a) **(10 points)** Obtain a point estimate of the proportion of the population who will respond to the flyer.
   **ANS:** *The point estimate is $\hat{p} = 5/50 = 0.1$*

   (b) **(10 points)** Obtain an 95% approximate score confidence interval for the population proportion estimated in (a).
   **ANS:** *Since $z_{0.975} = 1.96$,*

$$\tilde{p} = \frac{5 + 0.5 \cdot 1.96^2}{50 + 1.96^2} = 0.1285$$

   *The interval is then*

$$0.1285 \pm 1.96\sqrt{\frac{0.1285(1-0.1285)}{50}} = (0.0357, 0.2213)$$

   (c) **(10 points)** Explain what "95% confidence" means here.
   **ANS:** *Approximately 95% of all such intervals obtained in repeated sampling from the population will contain the true population proportion $p$.*

   (d) **(10 points)** Explain in layman's terms what the interval you computed in (b) means in terms of the population proportions involved.
   **ANS:** *We estimate that the proportion of the population that will respond to the flyers is between 0.0357 and 0.2213.*

   (e) **(10 points)** Suppose the market researchers want to estimate the population proportion to within $\pm 0.05$ with 95% confidence. For planning purposes, they are willing to assume that the true population proportion $p$ equals 0.10. How large a sample should they plan?
   **ANS:** $n = 1.96^2 0.10(1-0.10)/0.05^2 = 138.3$, *so take* $n = 139$.

3. A manufacturer of concrete beams analyzes the quality of the product by sampling randomly from production and measuring various characteristics. One of these is porosity of the concrete. Porosity measurements on 10 recently tested units give a mean porosity of 0.149 with standard deviation 0.017.

   (a) **(10 points)** Construct a level 0.90 confidence interval for the population mean porosity.
   **ANS:** *The interval is*

$$\overline{y} \pm \frac{s}{\sqrt{n}} t_{9,0.95} = 0.149 \pm \frac{0.017}{\sqrt{10}} 1.8331 = (0.139, 0.159)$$

   (b) **(10 points)** Construct an interval based on the 10 values in the data set which you are 90 percent confident will contain the porosity of the next unit tested. If the next sample tested has porosity 0.201, what will you conclude based on the interval you calculated?
   **ANS:** *We construct a level 0.90 prediction interval. The interval is*

$$\overline{y} \pm s\sqrt{1 + \frac{1}{n}} t_{9,0.95} = 0.149 \pm 0.017\sqrt{1 + \frac{1}{10}} 1.8331 = (0.116, 0.182)$$

*Since 0.201 is above the interval, we conclude that the porosity of this unit is greater than that of the population as represented by the 10 measurements in the sample.*

(c) **(10 points)** Construct an interval you are 90% confident will contain at least 99% of the porosities of all units in the population. Interpret this interval in terms of "90% confidence".

**ANS:** *We construct a level 0.90 tolerance interval for proportion 0.99 of the population values. The interval is*

$$\overline{y} \pm K \cdot s = 0.149 \pm 3.959 \cdot 0.017 = (0.082, 0.216)$$

*Interpretation: In repeated sampling, approximately 90% of all intervals constructed will contain at least 99% of all population values.*

(d) **(10 points)** For all these intervals, what is being assumed about the population distribution of porosity measurements?

**ANS:** *Normality is assumed.*