Test Your Understanding 0

The following table shows counts of vehicles passing a particular point on a road, grouped by day (columns) and hour (rows). There is something wrong with the table. Can you tell what it is?

<table>
<thead>
<tr>
<th>Day</th>
<th>Hours</th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thu</th>
<th>Fri</th>
<th>Sat</th>
<th>Sun</th>
</tr>
</thead>
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<td>5</td>
<td>15</td>
<td>19</td>
<td>35</td>
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<td>0100-0200</td>
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<td>5</td>
<td>5</td>
<td>3</td>
<td>8</td>
<td>10</td>
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<td>17</td>
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<td>0</td>
<td>4</td>
<td>6</td>
<td>9</td>
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<td>232</td>
<td>293</td>
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<td>299</td>
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<td>286</td>
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<td>301</td>
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<td>220</td>
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<td>377</td>
<td>393</td>
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<td>1900-2000</td>
<td>193</td>
<td>257</td>
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<td>242</td>
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<td>2200-2300</td>
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<td>86</td>
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</tbody>
</table>
Test Your Understanding 1

A process that produces audiotapes is monitored for the thickness of magnetic coating on the tapes. In Figure 1, the thicknesses of 150 thickness measurements (in microns) are plotted versus the order in which they were taken. Your boss asks you how the process is performing. What do you say?

Figure 1: One-hundred fifty consecutive measurements of the thickness of magnetic coating on audio tape
Test Your Understanding 2

Figure 2 shows a time series plot. What is the simplest moving average that will remove the cycles in the plot? Apply the moving average to the first 5 data values: $-0.95, 0.79, -0.99, 1.20, -1.03$. Plot the moving average values on the graph to demonstrate the cycles have been removed.

![Time series plot](image)

Figure 2: *Time series plot*
Test Your Understanding 3

A microhardness tester is a machine designed to test the hardness of material. In order to test the repeatability and reproducibility of the measuring process using a particular microhardness tester, four operators each took 20 measurements of the hardness of the same metal piece. Time series plots showed that the measuring process was stationary for each of them. Summarize what the stratified plot in Figure 3 tells about the R&R of the measuring process.

Figure 3: Force measurements from a microhardness tester
Test Your Understanding 4

Figure 3 shows four frequency histograms.

Figure 4: *Four frequency histograms*

Briefly describe the main features of each histogram.
Test Your Understanding 5

Figure 3 shows four frequency histograms.

![Histograms](image)

**Figure 5: Four frequency histograms**

For each histogram, describe the summary measures you would use to back up your description.
Test Your Understanding 6

The figure is a box and whiskers plot of the home run totals for 322 major league batters during the 1986 season.

Figure 6: Box and whiskers plot of home run totals for 322 major league batters during the 1986 season.

Which of the following statements about the data are supported by the plot? Explain.

(a) The data distribution is skewed right.

(b) There are no outliers in the data.

(c) The mean number of home runs is 8.
Test Your Understanding 7

Suppose you want to estimate the average amount spent by first term sophomores at WPI for textbooks, and that you can interview 10 students for your study.

(a) If you believe the distribution of the amounts spent for textbooks is pretty consistent across all students, how might you choose the 10 students? Why?

(b) If you believe that textbook expenses for engineering students are substantially higher than for other majors, how might you choose the 10 students? Why?

(c) If you want to be certain to obtain an estimate for humanities majors, as well as other majors, how might you choose the 10 students? Why?
Test Your Understanding 8

To compare the efficacy of mosquito repellant, volunteers have an arm coated with a prescribed amount of the product. The arm is then inserted into a chamber filled with mosquitoes for a fixed amount of time and the number of bites counted (YUCK!). To compare the efficacies of two different repellants, volunteers are randomly divided into two groups. One group is given repellant 1 and the other repellant 2 and the test described above is conducted for each.

1. Is this a controlled experiment? Why?

2. If it is a controlled experiment, describe the
   (a) Experimental units
   (b) Response
   (c) Experimental factor(s)
   (d) Possible nuisance factors
   (e) Factor levels
   (f) Treatments
   (g) Effect
Recall the experiment described in TYU 8:

To compare the efficacy of mosquito repellent, volunteers have an arm coated with a prescribed amount of the product. The arm is then inserted into a chamber filled with mosquitoes for a fixed amount of time and the number of bites counted (YUCK!). To compare the efficacies of two different repellants, volunteers are randomly divided into two groups. One group is given repellant 1 and the other repellant 2 and the test described above is conducted for each.

How could blocking be used to improve the design?
Test Your Understanding 10

In order to identify risk factors for juvenile criminal behavior, researchers compared a large group of juvenile offenders with a group of their peers who were not offenders. These groups were compared with respect to a large number of factors.

1. What kind of a study is this? Be as specific as you can.

2. Suppose the researchers find a number of factors that are quite different for the two groups. Do you think the researchers can conclude these factors cause juvenile crime? Explain.
Test Your Understanding 11

Graph the dynamic modulus (stress/strain, measured in mega-pascals) of a set of asphalt samples using a density histogram, with the intervals indicated.

<table>
<thead>
<tr>
<th>Range</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>[48, 54)</td>
<td>36</td>
</tr>
<tr>
<td>[54, 57)</td>
<td>51</td>
</tr>
<tr>
<td>[57, 67)</td>
<td>30</td>
</tr>
</tbody>
</table>
Test Your Understanding 12

A population histogram has four bars. The first corresponds to measurement value 1 and has area 0.40, the second corresponds to measurement value 3 and has area 0.25, the third corresponds to measurement value 8 and has area 0.30, and the fourth corresponds to measurement value 12.

(a) What is the area of the fourth bar?

(b) 1000 measurements are sampled randomly from the population. How many do you expect will have the value 1? 3? 8? 12? Why?
Test Your Understanding 13

Experience has shown that the width, in mm, of the flange on a plastic connector has the following distribution:

\[
\begin{align*}
p_Y(y) &= 50y, \quad 0.48 < y < 0.52, \\
&= 0, \quad \text{otherwise}
\end{align*}
\]

1. Of the next 1000 connectors produced, how many do you estimate will have widths between 0.50 and 0.51 mm? Show how you arrived at your estimate.

2. How many times as likely is it to produce connectors with flange width close to 0.51 mm as it is to produce connectors with flange width close to 0.49 mm? Justify your answer.
Test Your Understanding 14

A department at a college has 10 professors whose ages in years are 28, 44, 51, 32, 39, 48, 61, 55, 64, and 30. A random sample of size 4 is taken from this population, and the ages of those selected are 28, 48, 64, and 30. Compute the population mean \( \mu \), and the sample mean \( \overline{y} \).
Test Your Understanding 15

Suppose we sample 4 OJ containers from the production lot having population proportion \( p \) of acceptable containers. Calculate 
\[ P_Y(3) = P(Y = 3), \]
the probability of obtaining exactly 3 acceptable containers in the sample.
Test Your Understanding 16

A system consists of three identical components. The system can operate successfully only if at least two components are operating. The probability any one component lasts less than 100 hours is 0.06, and whether that component fails before 100 hours is independent of the performance of the other two components. If \( Y \) is the number of components in the system that fail before 100 hours,

(a) Obtain the distribution model of \( Y \).

(b) What is the probability the system fails before 100 hours?
Test Your Understanding 17

Suppose the population of math SAT scores follows a normal distribution with mean 500 and standard deviation 80. What proportion of students get between 600 and 700 on the exam?
Test Your Understanding 18

In Example 4.5, if 100 cereal boxes are used to compute the mean find the probability the estimate is within 0.1 ounces of the true mean weight.
Test Your Understanding 19

The state bar exam is designed so that 30% of prospective lawyers pass it each year, and over time, this passing percentage has held true, on average. From one year to the next, however, the percentage can vary. If 1000 prospective lawyers take the exam this year, and assuming each lawyer has a 0.30 probability of passing, approximately what is the probability that 320 or more pass?
Test Your Understanding 20

The following are the numbers of cycles to failure of 30 electrical appliances in a life test. The data are ordered for your convenience.

\[
\begin{array}{cccccccccccc}
14 & 59 & 69 & 123 & 165 & 381 & 479 & 574 & 917 & 991 \\
1088 & 1174 & 1275 & 1397 & 1578 & 1702 & 1932 & 2161 & 2326 & 2628 \\
2811 & 2993 & 3122 & 3715 & 3857 & 4100 & 4116 & 4510 & 5267 & 5583 \\
\end{array}
\]

The mean and standard deviation of the data are 2036.9 and 1646.7, respectively. Use the 68-95-99.7 rule to assess normality.
Test Your Understanding 21

Fill in the remainder of the table and create the normal quantile plot. Remember the data are 266, 149, 161, 220.

<table>
<thead>
<tr>
<th>k</th>
<th>((k - 0.375)/(n + 0.250))</th>
<th>(q(k))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.147</td>
<td>-1.05</td>
</tr>
<tr>
<td>2</td>
<td>0.382</td>
<td>-0.30</td>
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<td>3</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>4</td>
<td>.</td>
<td>.</td>
</tr>
</tbody>
</table>
Test Your Understanding 22

A data set consists of values 10, 12 and 23. It is believed that the C+E model is appropriate for these data, and it is desired to estimate the population mean, $\mu$, based on these data.

a. In terms of SSE, which of the two estimates, $\hat{\mu}_1 = 12$ or $\hat{\mu}_2 = 14$ is better?

b. What is the least squares estimate of $\mu$? Show that it has a better SSE value than the other two estimates.
20 tires of a new design give tread lives having a sample mean of 42,357 miles and a sample standard deviation of 3,042 miles. The data show no evidence of outliers or non-normality.

a. Obtain a 95% confidence interval for the true mean tread life.

b. From this interval can you conclude that mean tread life exceeds 42,000 miles? Why or why not?
Recall the problem from Test Your Understanding 23: Tests of 20 tires of a new design give tread lives having a sample mean of 42,357 miles and a sample standard deviation of 3,042 miles. The data show no evidence of outliers or non-normality.

a. Obtain a 95% prediction interval interval for a new observation.

b. A customer complains that he bought a set of four tires of this type and got only 38,000 miles on one of them. Based on your interval from part a, do you think this is a defective tire? Why?
Test Your Understanding 25

Ecologists want to estimate the proportion of deer in a certain region infected with the Lyme disease bacterium, *Borrelia burgdorfen*. To do so, they collect a random sample of 25 deer. Analysis shows that 6 of the deer are carrying the bacterium. Assuming the number of deer in the region is large, compute the approximate score level 0.90 confidence interval for the proportion of infected deer in the region.
Recall the problem from Test Your Understanding 23: *Tests of 20 tires of a new design give tread lives having a sample mean of 42,357 miles and a sample standard deviation of 3,042 miles. The data show no evidence of outliers or non-normality.*

Suppose we want to use a level 0.95 interval to estimate mean tread life to within 500 miles. We are willing to assume that $\sigma = 3000$. How large a sample will be needed?
Test Your Understanding 27

In an experiment to determine if there was a difference in gripping strength between the dominant and nondominant hand, 71 students measured the strengths of their grips with each hand. The mean of the differences between the dominant and nondominant hand measurements was 3.73 and the standard deviation was 4.83 pounds. Compute a 99% confidence interval to decide if there is a difference in mean grip between the dominant and nondominant hand. What do you conclude?
Test Your Understanding 28

Recall the data from Test Your Understanding 23: Tests of 20 tires of a new design give tread lives having a sample mean of 42,357 miles and a sample standard deviation of 3,042 miles. The data show no evidence of outliers or non-normality. A competitor introduces a new tire. In independent tests, 12 of the competitor's tires gave tread lives having a sample mean of 44,821 miles and a sample standard deviation of 2,568 miles. These data also show no evidence of outliers or non-normality. Assuming the population standard deviations are equal,

a. Obtain a 90% confidence interval for the difference of mean tire life for the two tire types.

b. Based on this interval, can you conclude the mean tire lives for the two tire types are different? Why?
Recall the statement in TYU 25: Ecologists want to estimate the proportion of deer in a certain region infected with the Lyme disease bacterium, *Borrelia burgdorfen*. To do so, they collect a random sample of 25 deer. Analysis shows that 6 of the deer are carrying the bacterium. Now suppose the ecologists want to compare the proportion of infected deer in this region with the proportion of infected deer in a distant region. They collect a random sample of 20 deer from the second region and find that 10 are infected.

Obtain a 99% approximate score interval for the difference of the proportions of infected deer in the two regions. Based on this interval, can you conclude there is a difference in the true population proportions? Why?
Recall the data from TYU 23: Tests of 20 tires of a new design give tread lives having a sample mean of 42,357 miles and a sample standard deviation of 3,042 miles. The data show no evidence of outliers or non-normality. Obtain an interval that with 90% confidence contains the tread life of at least 99% of this type of tire. Interpret the interval.