CHAPTER 6

HYPOTHESIS TESTS

6.2. (14 points: 2 for each part) For the normal approximation:

- **Scientific Hypothesis:** Clinton approval > 0.5.
- **Statistical Hypotheses:**
  \[ H_0 : \ p = 0.5 \]
  \[ H_a : \ p > 0.5, \]

  \( p \) = population proportion approving of Clinton.
- **Model:** \( b(10000, p) \).
- **Standardized Test Statistic:**
  \[ z^* = \frac{Y - np_0 - 0.5}{\sqrt{np_0(1 - p_0)}} = \frac{5100 - (10000)(0.5) - 0.5}{\sqrt{(10000)(0.5)(1 - 0.5)}} = 1.99 \]

  - **Assumptions:** \( Y, n - Y \geq 10 \). Satisfied since \( Y = 5100, \ n - Y = 4900 \).
  - **p-value:** \( P(Z \geq 1.99) = 0.0233, \ Z \sim N(0, 1) \)
  - **Conclusions:** Reject \( H_0 \) in favor of \( H_a \).

  For the exact test, change the following:
  - **Test Statistic:** \( y^* = 5100 \)
  - **Assumptions:** none needed
  - **p-value:** \( P(b(10000, 0.5) \geq 5100) = 0.0233 \).

6.6. (5 points) The one-sided test is appropriate since the complaints were that red came up too often. If the complaints were only that the wheel wasn’t fair, then a two-sided alternative hypotheses would be appropriate.

6.8. (14 points: 2 for each part)

- **Scientific Hypothesis:** Mean breaking strengths < 42,300
- **Model:** C+E one population.
- **Statistical Hypothesis:**
  \[ H_0 : \ \mu = 42,300 \]
  \[ H_a : \ \mu < 42,300, \]

  - **Standardized Test Statistic:**
    \[ t^* = \frac{\bar{y} - 42,300}{s/\sqrt{n}} = \frac{42166 - 42300}{259.41/\sqrt{5}} = -1.1551 \]

  - **Assumptions:** Normality.
  - **p-value:** \( P(t_4 \leq -1.1551) = 0.1562 \).
  - **Conclusions:** Do not reject \( H_0 \) in favor of \( H_a \).

6.14. (14 points: 2 for each part)

<table>
<thead>
<tr>
<th>Group</th>
<th>( \bar{y} )</th>
<th>s</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>40.7</td>
<td>10.8</td>
<td>50</td>
</tr>
<tr>
<td>Control</td>
<td>39.0</td>
<td>12.2</td>
<td>50</td>
</tr>
</tbody>
</table>
• **Scientific Hypothesis:** Refrigerated batteries last longer, on average.

• **Model:** Two population C+E

• **Statistical Hypothesis:**

\[
H_0 : \mu_T = \mu_C \\
H_a : \mu_T > \mu_C.
\]

\(\mu_T\) = mean life of refrigerated batteries

\(\mu_C\) = mean life of unrefrigerated batteries

• **Standardized Test Statistic:** Since the sample sizes are large, we compute

\[
z^* = \frac{\bar{y}_T - \bar{y}_C}{\sqrt{\frac{s_T^2}{n_T} + \frac{s_C^2}{n_C}}} = \frac{40.7 - 39.0}{\sqrt{\frac{110.8^2}{50} + \frac{12.2^2}{50}}} = 0.74
\]

• **Assumptions:** No extreme outliers.

• **p-value:** \(P(Z \geq 0.74) = 0.2297\)

• **Conclusion:** Do not reject \(H_0\).

6.20. (5 points) No. A large \(p\)-value signifies that the observed data are consistent with the null hypothesis, but it does not prove \(H_0\) to be true. Remember, in calculating the \(p\)-value, we assume \(H_0\) is true.

6.24. (14 points: 2 for each part)

• **Scientific Hypothesis:** The proportion of female students who prefer cookie A to cookie B is greater than the proportion of male students who prefer cookie A to cookie B.

• **Model:** The observed number of male students who prefer cookie A to cookie B is \(Y_1 \sim b(179, p_1)\), and the observed number of female students who prefer cookie A to cookie B is \(Y_2 \sim b(66, p_2)\). The populations are independent.

• **Statistical Hypothesis:**

\[
H_0 : p_1 = p_2 \\
H_a : p_1 < p_2.
\]

• **Standardized Test Statistic:** The point estimator of \(p_1\) is \(\hat{p}_1 = 94/179 = 0.53\), and of \(p_2\) is \(\hat{p}_2 = 39/66 = 0.59\). The standardized test statistic is

\[
z^* = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{0.53 - 0.59}{\sqrt{(0.54)(0.46)(\frac{1}{179} + \frac{1}{66})}} = -0.92,
\]

where

\[
\hat{p} = \frac{y_1 + y_2}{n_1 + n_2} = \frac{94 + 39}{179 + 66} = \frac{133}{245} = 0.54.
\]

• **Assumptions:** Since

\[
\begin{align*}
y_1 &= 94, & y_2 &= 39 \\
n_1 - y_1 &= 85, & n_2 - y_2 &= 27
\end{align*}
\Rightarrow all > 10,
\]

we are comfortable using the large sample test based on the normal approximation.

• **p-value:** \(P(Z \leq -0.92) = 0.1788\).

• **Conclusion:** Since 0.1788 > 0.05, do not reject \(H_0\). There is not enough evidence to conclude that the proportion of female students who prefer cookie A is greater than the proportion of male students who prefer cookie A.