CHAPTER 6

HYPOTHESIS TESTS

6.2. **(14 points: 2 for each part)** For the normal approximation:

- Scientific Hypothesis: Clinton approval > 0.5.
- Statistical Hypotheses:

$$\begin{array}{lcl} H_0: & p & = & 0.5 \\ H_a: & p & > & 0.5, \end{array}$$

  $p =$ population proportion approving of Clinton.
- Model: $b(10000, p)$.
- Standardized Test Statistic:

$$z^* = \frac{Y - np_0 - 0.5}{\sqrt{np_0(1-p_))}} = \frac{5100 - (10000)(0.5) - 0.5}{\sqrt{(10000)(0.5)(1-0.5)}} = 1.99$$

- Assumptions: $Y, n - Y \geq 10$. Satisfied since $Y = 5100, \quad n - Y = 4900$.
- p-value: $P(Z \geq 1.99) = 0.0233, \quad Z \sim N(0,1)$
- Conclusions: Reject H_0 in favor of H_a.

For the exact test, change the following:

- Test Statistic: $y^* = 5100$
- Assumptions: none needed
- p-value: $P(b(10000, 0.5) \geq 5100) = 0.0233$.

6.6. **(5 points)** The one-sided test is appropriate since the complaints were that red came up too often. If the complaints were only that the wheel wasn't fair, then a two-sided alternative hypotheses would be appropriate.

6.8. **(14 points: 2 for each part)**

- Scientific Hypothesis: Mean breaking strengths $< 42,300$
- Model: C+E one population.
- Statistical Hypothesis:

$$\begin{array}{lcl} H_0: & \mu & = & 42{,}300 \\ H_a: & \mu & < & 42{,}300, \end{array}$$

- Standardized Test Statistic:

$$t^* = \frac{\bar{y} - 42,300}{s/\sqrt{5}} = \frac{42166 - 42300}{259.41/\sqrt{5}} = -1.1551$$

- Assumptions: Normality.
- p-value: $P(t_4 \leq -1.1551) = 0.1562$.
- Conclusions: Do not reject H_0 in favor of H_a.

6.14. **(14 points: 2 for each part)**

Group	$\bar{y}$	s	n
Treatment	40.7	10.8	50
Control	39.0	12.2	50

- Scientific Hypothesis: Refrigerated batteries last longer, on average.
- Model: Two population C+E
- Statistical Hypothesis:

$$\begin{aligned} H_0: &\quad \mu_T = \mu_C \\ H_a: &\quad \mu_T > \mu_C. \end{aligned}$$

μ_T = mean life of refrigerated batteries
μ_C = mean life of unrefrigerated batteries

- Standardized Test Statistic: Since the sample sizes are large, we compute

$$z^* = \frac{\bar{y}_T - \bar{y}_C}{\sqrt{\frac{s_T^2}{n_T} + \frac{s_C^2}{n_C}}} = \frac{40.7 - 39.0}{\sqrt{\frac{(10.8)^2}{50} + \frac{(12.2)^2}{50}}} = 0.74$$

- Assumptions: No extreme outliers.
- p-value: $P(Z \geq 0.74) = 0.2297$
- Conclusion: Do not reject H_0.

6.20. **(5 points)** No. A large p-value signifies that the observed data are consistent with the null hypothesis, but it does not prove H_0 to be true. Remember, in calculating the p-value, we assume H_0 is true.

6.24. **(14 points: 2 for each part)**

- Scientific Hypothesis: The proportion of female students who prefer cookie A to cookie B is greater than the proportion of male students who prefer cookie A to cookie B.
- Model: The observed number of male students who prefer cookie A to cookie B is $Y_1 \sim b(179, p_1)$, and the observed number of female students who prefer cookie A to cookie B is $Y_2 \sim b(66, p_2)$. The populations are independent.
- Statistical Hypothesis:

$$\begin{aligned} H_0: &\quad p_1 = p_2 \\ H_a: &\quad p_1 < p_2. \end{aligned}$$

- Standardized Test Statistic: The point estimator of p_1 is $\hat{p}_1 = 94/179 = 0.53$, and of p_2 is $\hat{p}_2 = 39/66 = 0.59$. The standardized test statistic is

$$\begin{aligned} z^* &= \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}} \\ &= \frac{0.53 - 0.59}{\sqrt{(0.54)(0.46)(\frac{1}{179} + \frac{1}{66})}} \\ &= -0.92, \end{aligned}$$

where

$$\hat{p} = \frac{y_1 + y_2}{n_1 + n_2} = \frac{94 + 39}{179 + 66} = \frac{133}{245} = 0.54.$$

- Assumptions: Since

$$\left.\begin{aligned} y_1 = 94 &\quad y_2 = 39 \\ n_1 - y_1 = 85 &\quad n_2 - y_2 = 27 \end{aligned}\right\} \Rightarrow \text{all } > 10,$$

we are comfortable using the large sample test based on the normal approximation.

- p-value: $P(Z \leq -0.92) = 0.1788$.
- Conclusion: Since $0.1788 > 0.05$, do not reject H_0. There is not enough evidence to conclude that the proportion of female students who prefer cookie A is greater than the proportion of male students who prefer cookie A.