MA 2612 Test 1 B '03 Solutions

NAME: ______________________________

1. **(15 points)** It often happens that when an hypothesis test is conducted which rejects the null hypothesis in favor of the alternative at the usual significance level (such as 0.05), researchers say that "the alternative hypothesis is proved." Is it accurate to say this? Justify your answer.

   Solution: *Hypothesis tests do not prove or disprove hypotheses, because there is always the chance of a mistaken conclusion. For example, if a test is done at the* 0.05 *significance level, then even if H_0 is true, there is a 5% chance it will be rejected in favor of H_a.*

2. The marketing department of a large corporation wants to test the effectiveness of two advertizing methods in obtaining customer inquiries about a certain product. To do so, they contact a random sample of 20,000 potential customers by method 1 and receive inquiries from 237 potential customers, and contact a random sample of 10,000 potential customers by method 2 and receive inquiries from 159 potential customers.

   Conduct an hypothesis test, at the 0.01 level of significance, to compare the effectiveness of the two methods. Be sure to state the scientific hypothesis, the statistical model, the statistical hypotheses, the p-value, the outcome of the hypothesis test, and your conclusions. Be sure to show any needed assumptions are satisfied.

   (a) **(10 points)** The scientific hypothesis.
   Solution: *The two marketing methods differ.*

   (b) **(10 points)** The statistical model.
   Solution: *The two sample binomial model.*

   (c) **(10 points)** The statistical hypotheses being tested.
   Solution: $H_0 : p_1 - p_2 = 0$ *versus* $H_a : p_1 - p_2 \neq 0$, *where p_1 is the population proportion that would inquire about the product if contacted by method 1, and p_2 is the population proportion that would inquire about the product if contacted by method 2.*

   (d) **(10 points)** The standardized test statistic being used.
   Solution:

   $$Z = \frac{\hat{p}_1 - \hat{p}_2}{\hat{\sigma}_0(\hat{p}_1 - \hat{p}_2)},$$

   where $\hat{p}_1 = Y_1/n_1$ is the sample proportion from sample 1 that inquired and $\hat{p}_2 = Y_2/n_2$ is the sample proportion from sample 2 that inquired, and

   $$\hat{\sigma}_0(\hat{p}_1 - \hat{p}_2) = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)},$$

   for

   $$\hat{p} = \frac{Y_1 + Y_2}{n_1 + n_2}.$$

   (e) **(10 points)** The p-value.
   Solution: $\hat{p} = (237 + 159)/(20000 + 10000) = 0.0132$. *Therefore,*

   $$\hat{\sigma}_0(\hat{p}_1 - \hat{p}_2) = \sqrt{0.0132(1 - 0.0132)\left(\frac{1}{20000} + \frac{1}{10000}\right)} = 0.0013978.$$

   $\hat{p}_1 = 237/20000 = 0.01185$ $\hat{p}_2 = 159/10000 = 0.0159$ *Therefore, the observed value of Z is*

   $$z^* = \frac{0.01185 - 0.0159}{0.0013978} = -2.8974.$$

   Then $p_- = P(N(0,1) \leq -2.878) = 0.00188$, $p^+ = P(N(0,1) \geq -2.878) = 0.99812$, *so* $p\pm = 2min(0.00188, 0.99812) = 0.00376$.

(f) **(10 points)** The assumptions made.

Solution: *We assume the sample sizes are large enough to justify the normal approximation. We are confident this is the case since* $y_1 = 237$, $y_2 = 159$, $n_1 - y_1 = 19763$, *and* $n_2 - y_2 = 9841$ *all far exceed the 'rule of thumb' value of 10.*

(g) **(10 points)** Your conclusions.

Since $0.004 < 0.01$, *reject* H_0 *in favor of* H_a *at the 0.01 level of significance. Conclude that the populaiton proportions of responses to the two marketing methods differ.*

3. **(15 points)** A random sample from the C+E model has mean 2.457 and standard deviation 15.5. A large-sample test of

$$\begin{array}{llll} H_0: & \mu & = & 1 \\ H_a: & \mu & > & 1, \end{array}$$

gives a p-value 0.0024. What is the sample size?

Solution: *A p-value of* 0.0024 *corresponds to an observed* Z *value* $z^* = 2.82$. *This means that*

$$2.82 = z^* = \sqrt{n}\frac{\overline{y} - \mu_0}{s} = \sqrt{n}\frac{2.457 - 1}{15.5}.$$

Solving, we get $n = 900$.