Test Your Understanding 19

Each year a random sample of American high school seniors is surveyed to assess student attitudes. One question asked is the following: "On a scale of 1-5, with 1 being not important at all, and 5 being of the highest importance, how important is making a high salary in your future career?" The following table gives the results for three different years:

| Year | Number Surveyed | Mean |
|---|---|---|
| 1967 | 11 | 2.32 |
| 1982 | 11 | 3.78 |
| 1997 | 11 | 3.02 |

If the mean square error is 1.07, test the hypothesis that there is no difference in population mean response for the years 1967, 1982 and 1997. Conduct the test at the 0.05 significance level.

**Solution:**

$$\overline{y}_{\cdot\cdot} = \frac{(11)(2.32) + (11)(3.78) + (11)(3.02)}{11 + 11 + 11} = 3.04$$

*Then*

$$SSM = (11)(2.32 - 3.04)^2 + (11)(3.78 - 3.04)^2 + (11)(3.02 - 3.04)^2 = 11.73$$

*So that* $MSM = 11.73/(3-1) = 5.865$. *The observed* $F$ *statistic is then* $F^* = 5.865/1.07 = 5.4815$. *Under* $H_0$*, this has an* $F_{2,30}$ *distribution. From the table,* $F_{2,30,0.95} = 3.316$. *Since* $F^* > F_{2,30,0.95}$*, we reject* $H_0$.

Test Your Understanding 20

Recall the results of the survey of American high school seniors introduced in TYU 20. The question was: “On a scale of 1-5, with 1 being not important at all, and 5 being of the highest importance, how important is making a high salary in your future career?” The following table summarized the results for three different years:

| Year | Number Surveyed | Mean |
|---|---|---|
| 1967 | 11 | 2.32 |
| 1982 | 11 | 3.78 |
| 1997 | 11 | 3.02 |

The researchers want to compute approximate Tukey intervals for all pairwise comparisons at an overall 0.95 confidence level. If the mean square error is 1.07, obtain the interval for the difference of the 1997 and 1982 population means. Does the interval suggest a difference in the population means? Why?

**Solution:**

$$\hat{\sigma}(diff) = \sqrt{MSE\left(\frac{1}{11} + \frac{1}{11}\right)} = \sqrt{\frac{(1.07)(2)}{11}} = 0.441.$$

*Also,* $q_{0.95,3,30} = 3.49$*, so the interval is*

$$3.02 - 3.78 \pm 0.441\frac{3.49}{\sqrt{2}} = (-1.848, 0.328)$$

*The interval contains 0, so we cannot conclude there is a difference in the population means.*

Test Your Understanding 21

In order to evaluate the effectiveness of three non-invasive imaging methods for looking at tumors within the body, scientists measured the overall oxygen content of tumors in mice. A tumor was induced in each of 10 mice and the oxygen content of each tumor was measured using each of the three methods. As a " gold standard", the oxygen content of each tumor was measured invasively using a probe.

a. Explain why this is not a CRD.

   **Solution:** *Because each of the three methods are used on each experimental unit (mouse), it is not a CRD.*

b. Formulate an appropriate model for these data. Be sure to explain what the parameters mean.

   **Solution:** *An RCBD is an appropriate model. This will have the form*

$$Y_{ij} = \mu + \tau_i + \gamma_j + \epsilon_{ij},$$

   *where $\mu$ is the overall mean, $\tau_i$ is the efffect of treatment $i$, $\gamma_j$ is the effect of block (=mouse) $j$, and $\epsilon_{ij}$ is random error.*

Test Your Understanding 22

Consider the following ANOVA table for a RCBD.

| Analysis of Variance | | | | | |
|---|---|---|---|---|---|
| Source | DF | Sum of Squares | Mean Square | F Stat | Prob > F |
| Treatment | 6 | 1.32 | 0.22 | 3.38 | 0.0207 |
| Blocks | 3 | 2.28 | 0.76 | 11.69 | 0.0002 |
| Error | 18 | 1.17 | 0.065 | | |
| C Total | 27 | 4.77 | | | |

Write out the table that would result if we ignored the blocking. Do you think the blocking was effective?

**Solution:**

| Analysis of Variance | | | | | |
|---|---|---|---|---|---|
| Source | DF | Sum of Squares | Mean Square | F Stat | Prob > F |
| Treatment | 6 | 1.32 | 0.22 | 1.34 | 0.2838 |
| Error | 21 | 3.45 | 0.164 | | |
| C Total | 27 | 4.77 | | | |

*Yes, the blocking was effective, as it greatly reduced* MSE *and enabled us to find significant treatment differences.*