## MA3471

Name:

## Final Exam (closed book, closed notes) D Term, 2008

1. (15 points) Consider the ODE  $x'' + \sin(t)x' + 3x = 0$ . Please write an equivalent 1<sup>st</sup>-order system.

2. (10 points) What is the principle distinction between the hypotheses and conclusions of the two main existence theorems that we discussed in class, the Picard-Lindelof theorem and the Cauchy-Peano theorem.

- 3. (20 points) Please define/describe/state each of the following:
  - (a) Lipschitz continuous w.r.t. x
  - (b) autonomous ODE
  - (c) eigenfunction
  - (d) Hamiltonian system

- 4. (15 points) For the ODE  $x' + \cos(t)x = 1$ ,
  - (a) Please find an integrating factor.

(b) Please use your integrating factor from above to write the left-hand side of the ODE as an exact derivative.

(c) Please find the general solution of this ODE (keeping in mind that it may not be possible to evaluate certain integrals explicitly).

5. (15 points) Can the Arzela-Ascoli theorem be applied to the sequence of functions  $\{x_n(t) := \sin(nt)\}, 0 \le t \le \pi$ ? Explain your answer.

6. (15 points) Suppose that x' = A(t)x with

$$A(t) = \left[\begin{array}{cc} 2 & a(t) \\ b(t) & 4 \end{array}\right]$$

where a and b are periodic with period  $\pi$ . If  $\mu_1 = -1$  is one Floquet multiplier for this system, please find another Floquet multiplier. Also, what can be said about the stability of the trivial solution  $x \equiv 0$ ?

7. (10 points) Suppose  $x = \psi(t)$  is an orbit (trajectory) of the ODE x' = F(x) where  $F \in C(\mathbb{R}^n)$  and  $x : \mathbb{R} \to \mathbb{R}^n$ . What condition on a function V guarantees that V is decreasing on the orbits of the ODE, and how do you know?