MA3471 **Name:** ______________________________

Final Exam (closed book, closed notes) **D Term, 2010**

Show all work needed to reach your answers. You may use any theorem we discussed in class, but cite any theorem you use.

1. (30 points) Suppose $$A = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$$

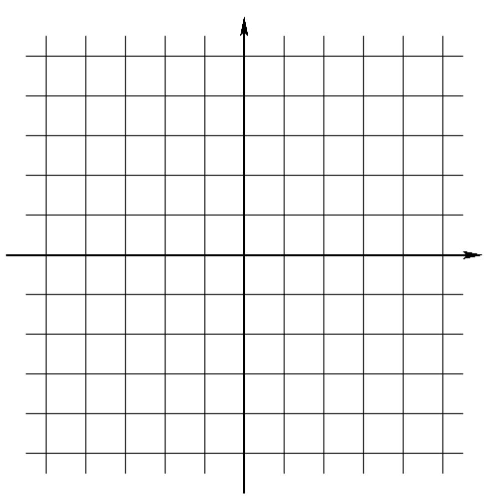
   (a) Please find the general solution for $x' = Ax$.

   General Solution: $x(t) =$ ______________________________

   (b) Please find a fundamental matrix for $x' = Ax$.

   Fundamental Matrix: $\Phi(t) =$ ______________________________

   (c) Please draw a rough sketch of the phase plane diagram for this system.



2. (15 points) Suppose that $x' = A(t)x$ with

$$A(t) = \begin{bmatrix} -2 & a(t) \\ b(t) & 1 \end{bmatrix}$$

where a and b are periodic with period 4π. If $\mu_1 = 1/4$ is one Floquet multiplier for this system, please find another Floquet multiplier. Also, what can be said about the stability of the trivial solution $x \equiv 0$?

3. (21 points) Consider the ODE $x' = -f(t, x)$ where $\forall\ t$, $f(t, x) > 0$ when $x > 0$; $f(t, x) < 0$ when $x < 0$; and $f(t, 0) = 0$. Suppose that the initial condition is $x(0) = 0$. So $x \equiv 0$ is *a* solution to this IVP.

   (a) If f is Lipschitz continuous w.r.t. x, what theorem guarantees that $x \equiv 0$ is the only solution?

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   (b) If f is *not* Lipschitz continuous w.r.t. x at $x = 0$, please explain why the solution $x \equiv 0$ is still unique.

   Proof: To see that $x \equiv 0$ is the unique solution, suppose $\exists\ t_1 > 0$ such that __________. Wolog (without loss of generality), suppose __________. Since x is differentiable on $(0, t_1)$, it must also be __________ on this interval, so let t_0 be the largest value of $t < t_1$ such that __________ and ______________________

   Notice that t_0 may be positive, but at a minimum, $t_0 = 0$.

   Now by ______________________ since x is ______________________

   $$x(t_1) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

   for some ______________________

   ______________________ $\square$

4. (24 points) Please define/describe/state each of the following:

   (a) Hamiltonian system

   (b) integrating factor

   (c) strict Lyapunov function

   (d) Strum-Liouville problem

5. (10 points) Suppose that $x(t)$, $y(t)$ and $z(t)$ satisfy the system

$$\begin{aligned} x' &= yz \\ y' &= -2xz \\ z' &= xy \end{aligned}$$

   What can be said about any solution to this system?
   Please explain.