**MA3471** **Name:** ____________________

**Final Exam (closed book, closed notes)** **D Term, 2012**

Show all work needed to reach your answers. You may use any theorem we discussed in class, but cite by name any theorem you use.

1. (10 points) For the ODE $x' + (4\cos t)x = \sin t$, please find an integrating factor $\mu(t)$ which can be used to make the left-hand-side an exact derivative.

Integrating Factor: $\mu(t) =$ ____________

2. (20 points) Suppose $$A = \begin{bmatrix} 2 & 1 \\ -1 & a \end{bmatrix}$$

   (a) For $a = -1$, what type of the equilibrium point is at the origin for the linear system $x' = Ax$.

   (b) Is there a value of $a$ such that origin is a *center* of $x' = Ax$? Please find such a value for $a$ or explain why it does not exist.

3. (20 points) Please define/describe/state each of the following:

(a) autonomous ODE

(b) sequence of equicontinuous functions $\{f_n(t)\}$

(c) Arzela-Ascoli theorem

(d) limit cycle

4. (10 points) Suppose that $A(t)$ is $\pi$-periodic. Please show that if $\mu = -1$ is a Floquet multiplier for $x' = A(t)x$, then there is a nontrivial solution with period $2\pi$.

5. (15 points) Consider the BVP: $\epsilon y'' + yy' + y^3 = 0$, $y(0) = 1$, $y(1) = 3$.

   (a) Please use dominant balance to find the correct inner variable $X = x/\epsilon^\alpha$ for a boundary layer region near $x = 0$.

   (b) Please find the zeroth-order inner equation.

6. (15 points) Please find constants $\alpha$ and $\beta$ so that $V(x, y) := \alpha x^2 + \beta y^2$ defines a Liapunov function for the system

$$\begin{aligned} x' &= -x - 5y \\ y' &= 3x - y^3 \end{aligned}$$

7. (10 points) For an interval $[a, b] \in \mathbb{R}$, suppose that $p, q \in C[a, b]$. Suppose that $x_1$ and $x_2$ are both solutions of $(p(t)x')' + q(t)x = h(t)$ satisfying $x(a) = 0$, $x(b) = 0$. What can be said about the Wronskian $W[x_1, x_2]$? Please explain your answer.