MA3471 Name: Solutions

Quiz 1 D Term, 2010

Show all work needed to reach your answers.

1. (10 points) Suppose $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$

6 (a) Please find the general solution of $x' = Ax$.

$tr(A) = 0$ & $det(A) = -1 \Rightarrow \lambda_1 = -1, \lambda_2 = 1$

$\vec{v}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ since $A - \lambda_1 I = \begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix}$

$\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ since $A - \lambda_2 I = \begin{bmatrix} 1 & -1 \\ 3 & -3 \end{bmatrix}$

$\Rightarrow x(t) = c_1 e^{-t} \begin{bmatrix} 1 \\ 3 \end{bmatrix} + c_2 e^{t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

4 (b) Please find a fundamental solution for $x' = Ax$.

$$\Phi(t) = \begin{bmatrix} e^{-t} & e^{t} \\ 3e^{-t} & e^{t} \end{bmatrix}$$

5 2. (10 points) Consider the set $\{\varphi_1(x), \varphi_2(x), ..., \varphi_n(x)\}$ defined for $x \in \mathbb{R}$.

(a) Please state the definition of *linearly independent.*

$\{\varphi_1, \dots, \varphi_n\}$ is L.I. iff $c_1\varphi_1 + c_2\varphi_2 + \dots + c_n\varphi_n \equiv 0 \ \forall x$

$\Rightarrow c_1 = c_2 = \dots = c_n = 0.$

5 (b) If $\varphi_n(x) = e^{n-x}$ for $n = 1, 2, 3, ..., 10$, is the set $\{\varphi_1(x), \varphi_2(x), ..., \varphi_{10}(x)\}$ linearly independent or linearly dependent? Please explain your answer.

Since $\varphi_n = e^n e^{-x}$, each φ_n is a constant multiple of e^{-x}. Thus the φ_n are linearly dependant.