MA3471 Name: Solutions

Quiz 3 D Term, 2010

Show all work needed to reach your answers. You may use any theorem we discussed in class, but cite any theorem you use.

1. (10 points) Consider the system of two ODEs

$$\begin{aligned} x' &= 4x + y - x(x^2 + y^2) \\ y' &= -x + 4y - y(x^2 + y^2) \end{aligned}$$

(a) (a) Please the **radial** equation for the polar coordinate version of this system. Hint: Notice that since $r^2 = x^2 + y^2$, $rr' = xx' + yy'$

So $rr' = xx' + yy' = x(4x+y-x(x^2+y^2)) + \cancel{x}(-x+4y-y(x^2+y^2))y'$ +2

$= 4(x^2+y^2) - (x^2+y^2)(x^2+y^2)$ +2

$= 4r^2 - r^4$

$\Rightarrow r' = r(4-r^2)$ +1

(b) (b) What is a limit cycle for this system, and is it stable or unstable? Please explain your answer. The limit cycle is +2 $r=2$, and it is stable, +1 because $r'>0$ for $r<2$ while $r'<0$ for $r>2$. +2

2. (10 points) What are the similarities and differences in the hypotheses and conclusions of the two main existence theorems for an IVP ($x' = f(t,x), x(t_0) = x_0$) that we discussed in class: the Picard-Lindelof theorem and the Cauchy-Peano theorem?

The Picard-Lindelof theorem requires that $f \in LC(I)$ for some interval I containing t_0, and it yields that there is a unique solution to the IVP. The Cauchy-Peano theorem, on the other hand, only requires that $f \in C(I)$, but it does not guarantee uniqueness, only existance of at least one solution.