MA3831 **Name:** ____________________ **Section:** ______

Final (Open Book, Open Notes) **A Term, 2011**

If you are/were a WPI undergrad, what was your first math course at WPI? ____________

This is an open book/open notes exam. Show all work needed to reach your answers. You may use any theorem proven in our text or on the compactness sheet, but cite by page number or name any theorem that you use.

1. (20 points) Suppose that (E, d) is a metric space, and let $G \subset E$ be *open*. Please make four distinct statements about G that are equivalent to "open".

   (a) ____

   ____

   (b) ____

   ____

   (c) ____

   ____

   (d) ____

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2. (20 points) For $S \subset \mathbb{R}^n$, suppose that S is bounded. Further suppose that $F \subset S$ and that F is closed. Please show that F is compact.

3. (20 points) Please explain precisely why $\mathbb{Q}$ is incomplete.

4. (25 points) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined as

$$f(x,y) = \begin{cases} \dfrac{xy^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Please discuss as completely as possible the continuity of this function at all points $(x,y) \in \mathbb{R}^2$. If the function f is continuous at a point, prove that f is continuous. If f is discontinuous at a point, please explain why f is discontinuous. **Hint**: Consider separately two cases: (1) when $(x,y) = (0,0)$ and (2) when $(x,y) \neq (0,0)$.

5. (15 points) Assume that $A \subset \mathbb{R}$ is non-empty. Furthermore assume that every increasing and bounded sequence in A is convergent in A. Please give a counterexample to disprove the statement "A is compact", and explain why your counterexample works.